

Coding of contrasts and interactions in non-experimental research

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1 Introduction

The following textbook is concerned with the estimation of effects from categorical and continuous variables and its interactions on **one** continuous variable in linear models. What will be said also holds, in principle, for non-linear probability models, although things become more difficult as far as predicted probabilities instead of logits are concerned. We will describe and explain several coding schemes with and without interactions and demonstrate how to interpret the model parameters. This guideline seems to be necessary, because information about how to construct design matrices adequately is not easy to be obtained. On the contrary, it is dotted about here and there in books and articles employing different notation, making it hard to understand for the non-technical reader and/or applied researcher. The main goal of this booklet therefore is to demonstrate the applicability of several of these schemes for non-experimental research. The focus is on the interpretation of the parameters resulting from a more or less complex design, employed as a matrix of predictors. Problems of statistical inference will only be allusively addressed since this is not the main topic of this small textbook. Nevertheless, the estimation of standard errors for combination of parameters will be addressed and outlined briefly.

1.1 *A general overview*

This guideline is an attempt to bridge the gap between what can be found in books and articles about Variance/Covariance analysis, and in texts on linear regression models. In most instances the focus is on testing, and the interpretation of more complex combination of parameters is badly neglected. We also want to show that the very common distinction between continuous and categorical predictors is entirely vacuous. For a linear model all the predictors are treated “as is”, which means they are treated as vectors of real numbers, and it is up to the researcher to take care of what these figures actually portray. Or, the other way round, one must establish a set of vectors which decompose the dependent variable into a set of expected values such mapping the hypotheses under consideration.

Four different coding schemes will be presented and evaluated with suitable examples. Dummy coding, effect coding, forward adjacent coding and Helmert coding. In the final chapter a way to compute ones own contrasts is presented, first by means of how to define one of the already illustrated contrasts, and secondly by an arbitrary scheme not predefined by one of the contrasts available from the software used throughout this textbook. For sake of

space we will not present more than one example for each of the contrast schemes, rather the reader is asked to repeat a particular example using the other dataset provided.

The reader should bear in mind that this text was not written to serve as a substitute for comprehensive and far more detailed books on linear models, like those cited in the text below. Important topics like orthogonality of a design and all the problems of multiple testing, for instance, will not be addressed at all. Additionally, problems arising from non-balanced designs are not treated. Several corrections outlined, for instance, by Pedhazur (Pedhazur, 1982) will not be described in the following pages. All this, and a lot of even more important topics can be found in the literature about ANOVA/ANCOVA (to mention only two: Rabe-Hesketh & Everitt, 2004; Rencher, 2000). However, it might serve as a compendium on how to construct and interpret more complex designs derived from theoretical assumptions.

1.2 Data

We will present the empirical examples using data from two different studies:

1. A study on attitudes towards the mentally ill was conducted in Germany in 2001 (N=5025). As a dependent variable we employ an index to measure the expected social distance towards a person described by means of a cases story. This index is the sum of 7 items regarding 7 situations, from neighbourhood to marriage. For each of them the respondent is asked to express his/her opinion concerning their willingness to accept the person described in the cases-story. The item-format is of the Likert type from 0 (definitely accept) to 4 (definitely reject). The sum therefore covers a range from 0 to 28. This score will be called `dist1sum`. To identify the person, two different case stories are presented by means of a so called “vignette”. Each respondent is presented only one of them, which results in a dichotomous variable called `q0rec` in the following. The first case-story gives a sketchy description of a schizophrenic episode (coded 0), the second one present the symptoms of a major depression (coded 1). Both stories can be found in the appendix. For a second categorical predictor we employ a variable, resulting from an open question, where respondent were asked to classify the “vignettes” just outlined above. Responses were classified into four categories: 1: The respondent classifies the story correctly either as schizophrenia or depression. 2: The respondent labelled the person described at least as mentally ill. 3: The person described is labelled aberrant or ill. 4: A residual

category comprising all other answers. We will employ different forms of collapsing to 3 categories or to a dichotomous variable. Variables will be called `skt` (4 cat.), `skt3` (3 cat.) or `sktdrec` (dichotomous).

Table 1: `tab skt sktdrec`

skt	skt3			sktdrec		Total
	1	2	3	0	1	
1.00	1,511	0	0	1,511	0	1,511
2.00	1,816	0	0	1,816	0	1,816
3.00	0	995	0	0	995	995
4.00	0	0	703	0	703	703
Total	3,327	995	703	3,327	1,698	5,025

- The second study we adopt for presenting the examples is the ESEMED study in 6 countries (N=21425). We employ the mental health score (`mcs12`) of the SF36 as dependent variable. For predictors we use the 6 countries (coded by means of several different schemes) and the dichotomous partition of the sample into non-depressive and depressive respondents respectively by means of `dsm_mde`.

We will not go into further details regarding the goals of these studies, nor will we describe or critically evaluate the characteristics of samples and variables. All interpretations of results should be considered cautiously and do not claim exclusivity. The reader is encouraged to apply the following examples to his/her own problems with his/her own data sets. Data sets of the two studies will be provided.

1.3 Software tools

Although each contrast for virtually all the models may be estimated by any statistical package, we will focus on STATA (StataCorp, 2005) and one of the pre-commands for STATA named `xi3`. Furthermore we will make use of the command `estimates` and its options in order to store several models in memory and compare them by means of likelihood ratio tests (`lrtest`). In order to test sets of parameters or linear combination of parameters, we will make use of the command `test` or `lincom` respectively. We also will make use of the matrix facilities of STATA, particularly to estimate the standard errors of parameters not immediately provided by a particular regression model. To compute and draw conditional and partial effects (slopes) we also employ the program `postgr3`, a post estimation command

which only works in conjunction with the `spostado` suite provided by Jeremy Freese and Scott Long (Long & Freese, 2003). STATA user can implement these tools for free by typing:

```
net from http://www.indiana.edu/~jlsoc/stata
net install spostado
```

Most of the figure and table captions will provide the necessary STATA commands by which the graph models were generated.

2 Dummy coding

By “dummy coding” we mean that each level of a categorical variable will be compared to a reference category level (also called “omitted category”). Variables or indicator variables should be coded zero and one, since then the effect parameter displays the exact mean difference between the omitted category and the category coded 1. Employing a predictor coded zero and one, and thereby distinguishing between two groups of observations is equivalent to a t-test. If the variables are coded, for instance, 0 and 2, the actual mean difference is twice the regression parameter, and the t-value remains the same, because the standard error is now also half of the standard error of the (0,1) coded predictor. If the variable is coded 1 and 2 things will become more complicated and will not be addressed further more. To cut the long story short: To estimate mean differences by dummy coding, always adopt the (0,1) coding scheme. Nevertheless we do present the results of both codings below. In the first example we employ `dist1sum` as dependent variable and `sktdrec` as predictor. The goal of this analysis is to find out whether the two subgroups differ with respect to their means on the social distance scale.

2.1 One dichotomous predictor

Table 2: `regress dist1sum sktdrec`

Source	SS	df	MS			
Model	2049.24049	1	2049.24049	Number of obs =	4976	
Residual	210214.383	4974	42.2626424	F(1, 4974) =	48.49	
Total	212263.624	4975	42.666055	Prob > F =	0.0000	
				R-squared =	0.0097	
				Adj R-squared =	0.0095	
				Root MSE =	6.501	

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
<code>sktdrec</code>	-1.355258	.1946273	-6.96	0.000	-1.736813	-.9737026
<code>_cons</code>	17.25342	.113391	152.16	0.000	17.03113	17.47572

The tabulation of means for both categories of `sktdrec` shows, that we obtain a test for the differences of means. A t-test will, of course, provide exactly the same results. Since we have only one predictor the square root of the F(1,4974)- value is exactly the t-value.

Table 3: `tab sktdrec, sum(dist1sum)`

Summary of egen			
dist1sum=rsum(q15a-q15g) if			
distmiss==0			
sktdrec	Mean	Std. Dev.	Freq.
0	17.253423	6.4038822	3287
1	15.898165	6.6859336	1689
Total	16.793408	6.5319258	4976

Coding the predictor 0 and 2 instead of 0 and 1, we obtain the same value for the constant (the omitted category) but the effect (now called `sktdrec02`) is only half of the actual mean difference we already know from Table 2 (compare the figures in bold face). This is necessarily so, since the regression equation reads:

$$\hat{y} = _cons + sktdrec02 * 0 \quad \text{for the omitted category, and}$$

$$\hat{y} = _cons + sktdrec02 * 2 \quad \text{for the second category.}$$

Table 4: `regress dist1sum sktdrec02`

Source	SS	df	MS	Number of obs = 4976		
Model	2049.24049	1	2049.24049	F(1, 4974) =	48.49	
Residual	210214.383	4974	42.2626424	Prob > F =	0.0000	
Total	212263.624	4975	42.666055	R-squared =	0.0097	
				Adj R-squared =	0.0095	
				Root MSE =	6.501	

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
sktdrec02	-.677629	.0973137	-6.96	0.000	-.8684067	-.4868513
_cons	17.25342	.113391	152.16	0.000	17.03113	17.47572

The real mean difference is now two times the parameter, the standard error of the regression parameter is exactly half of what is observed if 0/1 coding is adopted. Therefore inference remains invariant; the constant also still represents the mean with respect to the omitted category (definition of to be mentally ill). Further on we will never use such a coding system; it simply serves to justify the usual 0/1 coding to estimate simple mean differences. Of course, this scheme holds even for predictors with more categories, since the multicategory variable is then decomposed into several dichotomous variables, as will be shown in section 2.3.

Study suggestions 1: Try to estimate the regression coding the predictor 2 & 4. Interpret both the effect parameter and the constant.

2.2 Two dichotomous predictors

For the next example we again use the social distance score (`dist1sum`) as the dependent continuous variable. The type of the vignette questionnaire `q0` and `skt` (4 categories of labelling) is used as independent variables. This variable discriminates between those respondents who label the person described in the vignette at least as mentally ill, and those who did not (comp. Section 1.2.). With the STATA command: `tab sktdrec q0rec, sum(dist1sum)` you will obtain a 2x2 table displaying the means of the dependent variable (`dist1sum`) within the different levels of the independent variables.

Table 5: Distance to people with mental disorders (`dist1sum`, mean scores) by `q0rec` and `sktdrec`

Sktdrec	q0rec		Total
	0	1	
0	19.200808 y_{11}	15.081725 y_{12}	17.253423
1	17.116022 y_{21}	14.984456 y_{22}	15.898165
Total	18.586488	15.044462	16.793408

By means of the command: `regress dist1sum q0rec sktdrec` we obtain the following regression coefficients (Table 6) and the estimated means for each of the four cells (Table 7). Results are stored under the name `mo1`.

Table 6: `regress dist1sum q0rec sktdrec`

Source	SS	df	MS	Number of obs = 4976		
Model	16744.0457	2	8372.02287	F(2, 4973) = 212.94		
Residual	195519.578	4973	39.3162232	Prob > F = 0.0000		
Total	212263.624	4975	42.666055	R-squared = 0.0789		
				Adj R-squared = 0.0785		
				Root MSE = 6.2703		

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
q0rec	-3.452283	.1785708	-19.33	0.000	-3.80236	-3.102205
sktdrec	-1.014958	.1885438	-5.38	0.000	-1.384587	-.6453289
_cons	18.88556	.1381608	136.69	0.000	18.61471	19.15642

```
predict dist1sump if e(sample)
```

```
(option xb assumed; fitted values)
```

```
(49 missing values generated)
```

```
estimates store mol
```

Table 7: `tab sktdrec q0rec, sum(dist1sump)`

sktdrec	q0rec		Total
	0	1	
0	18.885563	15.433281	17.253422
	.	0	1.7238418
	1733	1554	3287
1	17.870605	14.418323	15.898164
	0	.	1.7089851
	724	965	1689
Total	18.586487	15.044462	16.793408
	.46280669	.49350943	1.8345671
	2457	2519	4976

If this model would suffice, the mean differences between the categories of one predictor are equal for both categories of the other:

1. $y_{11} - y_{12} \mid X_2 = 0$
2. $y_{21} - y_{22} \mid X_2 = 1 = \mathbf{b}_1$

in terms of the additive linear combination of parameters :

$$\begin{aligned}(c + b_1 \cdot 0 + b_2 \cdot 0) - (c + b_1 \cdot 1 + b_2 \cdot 0) &= \mathbf{-b_1} \\(c + b_1 \cdot 0 + b_2 \cdot 1) - (c + b_1 \cdot 1 + b_2 \cdot 1) &= \mathbf{-b_1}\end{aligned}$$

Of course, this also holds for X_1 .

The model above is not saturated – the means for the 4 cells are estimated by only 3 parameters – and the estimated means do not equal the real means (Table 5 and Table 7). Therefore we will immediately turn to the fully saturated model, which model not only the differences for each predictor, but also the dependence of each effect on the different values of the other predictor. Here we use a pre-command feature of STATA: `xi3` for the first time, which helps to built special contrasts and interactions. We did not employ this feature up to now, since both predictors are already coded 0/1. To get an interaction term, a simple star is inserted. If this command (`xi3`) is not available the interaction variable has to be computed

beforehand by simply setting up the product of `q0rec` and `sktdrec`, generating a new variable. This model is stored under the name `mo2`.

Table 8: `xi3:regress dist1sum q0rec*sktdrec`

Source	SS	df	MS			
Model	17829.863	3	5943.28765	Number of obs =	4976	
Residual	194433.761	4972	39.1057443	F(3, 4972) =	151.98	
Total	212263.624	4975	42.666055	Prob > F =	0.0000	
				R-squared =	0.0840	
				Adj R-squared =	0.0834	
				Root MSE =	6.2535	

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
q0rec	-4.119083	.2184718	-18.85	0.000	-4.547384	-3.690782
sktdrec	-2.084786	.2767288	-7.53	0.000	-2.627296	-1.542275
_Iq0Xsk	1.987517	.3771836	5.27	0.000	1.248071	2.726963
_cons	19.20081	.1502177	127.82	0.000	18.90631	19.4953

`estimates store mo2`

To calculate the means of the dependent variable (Table 5), one has to use the equation for the linear regression model and the regression coefficients from the STATA output of the

$$\hat{y} = \text{cons} + b1*q0rec + b2*sktdrec + b3*(q0rec*sktdrec)$$

The precommand `xi3` will produce a variable called `_Iq0Xsk` which represents the product of `q0rec` and `sktdrec`. We should have a closer look at how the cell means are generated by linear combinations of the model parameters. If, for instance, you insert into the equation the following values you will get:

$$y_{11} = \text{cons} + b1 (q0rec=0) + b2 (sktdrec=0) + b3 (q0rec=0*sktdrec=0) =$$

$$\text{cons} + b1*0 + b2*0 + b3*0 = \mathbf{\text{cons} = 19.200808}$$

This cell represents the constant in the regression model, since both independent variables are coded zero (`sktdrec=0` and `q0rec=0`). Therefore, this cell mean represents the mean for **both** reference categories.

$$y_{21} = \text{cons} + b1 (q0rec=0) + b2 (sktdrec=1) + b3 (q0rec=0*sktdrec=1) =$$

$$\text{cons} + b1*0 + b2*1 + b3*0 = \mathbf{\text{cons} + b2}$$

$$y_{21} = 19.200808 + (-2.084786) = \mathbf{17.116022}$$

The mean in this cell represents the mean for people with $q0rec = 0$ and $sktdrec = 1$. To obtain this cell mean, you have to add the difference between both levels of $sktdrec$ (represented by the coefficient of $sktdrec = -2.084786$) to the constant.

$$y_{12} = \text{cons} + b1 (q0rec=1) + b2 (sktdrec=0) + b3 (q0rec=1*sktdrec=0) = \text{cons} + b1*1 + b2*0 + b3*0 = \mathbf{\text{cons} + b1}$$

$$y_{12} = 19.200808 + (-4.119083) = \mathbf{15.081725}$$

The mean in this cell represents the mean for people with $q0rec = 1$ and $sktdrec = 0$. To obtain this cell mean you have to add the difference between both levels of $q0rec$ (represented by the coefficient of $q0rec = -4.119083$) to the constant.

$$y_{22} = \text{cons} + b1 (q0rec=1) + b2 (sktdrec=1) + b3 (q0rec=1*sktdrec=1) = \text{cons} + b1*1 + b2*1 + b3*1 = \mathbf{\text{cons} + b1 + b2 + b3}$$

$$y_{22} = 19.200808 + (-4.119083) + (-2.084786) + 1.987517 = \mathbf{14.984456}$$

The mean in this cell represents the mean for people with $q0rec = 1$ and $sktdrec = 1$. To obtain this cell mean you have to add the difference between both levels of $q0rec$ (represented by the coefficient of $q0rec = -4.119083$) and the difference of both levels of $sktdrec$ (represented by the coefficient of $sktdrec = -2.084786$) as well as the coefficient of the interaction term ($_{I}q0Xsk = 1.987517$) to the constant. In Table 9 the more general way of reconstructing the cell means from a combination of the 4 parameters is demonstrated.

Table 9: Decomposition table

		q0rec	
		0	1
sktdrec	0	cons Y_{11}	cons + b1 Y_{12}
	1	cons + b2 Y_{21}	cons + b1 + b2 + b3 Y_{22}

In the case of a saturated model with interaction (Table 8), the so called “simple slopes” (printed in bold face) are always additive linear combinations of the following form:

$$\hat{Y} = \text{cons} + (\mathbf{b1} + \mathbf{b3 sktdrec}) q0rec + b2sktdrec \text{ or:}$$

$$\hat{Y} = \text{cons} + (\mathbf{b2} + \mathbf{b3 q0rec}) sktdrec + b1sktdrec$$

We see that the simple effects depend on both the value of the interaction term and the value of the predictor which is weighted by the interaction term (here b_3). The coefficients b_1 or b_2 portray the effect of $q0rec$ or $sktdrec$ only if the other variable **or the coefficient b_3** is zero. This simple relation always applies, whether the predictors are categorical (dichotomous or multicategorical) and therefore coded in one or the other way, or continuous, where we have to take care about the **meaning** of zero. Of course, we could also carry out a likelihood ratio test (`lrtest`) to compare the restricted model with the fully saturated one. Taking the square root of the χ^2 value 27.71, we get 5.264, which is, apart from rounding errors, the t-value from the regression analysis, found in Table 8. Strictly speaking, this `lrtest` is not necessary if only one parameter is tested. To administer this test, the two models are stored in memory immediately after the estimation by the command `estimates store [modelname]`. For `[modelname]` we adopted, as said before, `mo1` and `mo2`.

Table 10: `lrtest mo1 mo2`

```
Likelihood-ratio test                                LR chi2(1) =      27.71
(Assumption: mo1 nested in mo2)                    Prob > chi2 =      0.0000
```

2.3 Two predictors (one dichotomous, one three categorical variable)

Now we employ the dichotomous predictor ($q0rec$) and $skt3$ as independent variables to estimate the `dist1sum` score. We collapsed the first two categories of skt (correct labelling and mentally ill) in order to get a 3-categorical variable. Let's first have a look at the 3x2 table, displaying the means of the dependent variable (`dist1sum`) for the different levels of the independent variables.

Table 11: `tab skt3 q0rec, sum(dist1sum)`

skt3	q0rec		Total
	0	1	
1	19.200808 y_{11}	15.081724 y_{12}	17.253423
2	16.798761 y_{21}	14.694611 y_{22}	15.380424
3	17.371571 y_{31}	15.636364 y_{32}	16.633237
Total	18.586488	15.044462	16.793408

Since the categorical variable `skt3` has three levels, we will have to use two indicator variables (`I1` and `I2`) to represent the categories of `skt3`. In all of the coding systems presented you need to create $k-1$ variables (where k represents the number of levels of the categorical variable). The remaining level of the categorical variable is redundant. Again, we do not need a new variable for representing the case-story, since it is already coded 0 and 1.

Table 12: Indicator variables for `skt3` if category 3 serves as reference category:

<code>skt3</code>	I1	I2
1	1	0
2	0	1
3	0	0

Using the last category as reference, of course, is an arbitrary decision and the reader is invited to estimate and interpret models with other “omitted” categories. The following regression analysis (Table 13) shows the parameter estimates of a model without interaction.

Table 13: `xi3:regress dist1sum q0rec i.skt3`

```

i.skt3          _Iskt3_1-3          (naturally coded; _Iskt3_3 omitted)
-----+-----
Source |           SS          df           MS          Number of obs =    4976
-----+-----
Model  |  16809.1834           3    5603.06113      F( 3, 4972) = 142.53
Residual | 195454.44    4972    39.3110298      Prob > F      = 0.0000
-----+-----
Total  | 212263.624    4975    42.666055      R-squared     = 0.0792
                                           Adj R-squared = 0.0786
                                           Root MSE    = 6.2699
-----+-----
dist1sum |           Coef.      Std. Err.      t    P>|t|     [95% Conf. Interval]
-----+-----
q0rec   | -3.419012          .1804199    -18.95  0.000     -3.772715     -3.06531
_Iskt3_1 |  .7818018          .2614418     2.99   0.003     .2692605     1.294343
_Iskt3_2 | -.4029666          .3130475    -1.29   0.198    -1.016678     .2107446
_cons   | 18.08803           .2494253    72.52   0.000     17.59905     18.57702
-----+-----

```

`estimates store m01`

```

predict dist1sump3 if e(sample)
(option xb assumed; fitted values)
(49 missing values generated)

```

The estimated means of the criterion, assuming that all the mean differences with respect to the omitted category are equal for both values of `q0rec` are provided in Table 14.

Table 14: `tab skt3 q0,sum(dist1sump3)`

Means, Standard Deviations and Frequencies of Fitted values

skt3	Schizo	Depress	Total
1	18.869835	15.450822	17.253423
	.	2.771e-06	1.7072295
	1733	1554	3287
2	17.685066	14.266053	15.380424
	0	.	1.6033776
	323	668	991
3	18.088032	14.66902	16.633237
	.	.	1.6916361
	401	297	698
Total	18.586488	15.044462	16.793408
	.45175368	.52847	1.8381326
	2457	2519	4976

If we compare the estimated means of this non-saturated model with the actual means, we see immediately that these figures differ considerably. However, we should interpret the regression parameter `q0rec` as the difference for social distance between the two case-stories, and this difference must be the same for all categories of `skt3`, since we have no interaction. This relation must also hold for the two dummy variables `_Iskt3_1` and `_Iskt3_2`: They represent the difference between the omitted category (3) and one of the other cells. These differences are the same for both columns. But, as said before, the predicted means are quite different from the actual means, and it becomes necessary to estimate the fully saturated model, including two additional (interaction-) effects in order to parameterise the conditional differences for category 1 & 2 of `skt3` with regard to `q0rec`. Of course, the estimated means will then be equal to the real means of each cell, just as in section 2.2. We obtain the following results using the pre-command `xi3` (Table 15).

Both the model without and with interaction effects have been stored with the names `mo1` and `mo2` respectively. The likelihood ratio test clearly shows that the interaction effects are statistically indispensable and we should only accept the saturated model. To calculate the means of the dependent variable shown in Table 11, you have to use the equation (including the indicator variables) for the linear regression model and the obtained coefficient from the STATA output of the `regress` command.

Table 15: `xi3:regress dist1sum q0rec*i.skt3`

```

i.skt3          _Iskt3_1-3          (naturally coded; _Iskt3_3 omitted)

-----+-----
Source |           SS          df           MS          Number of obs =      4976
-----+-----+-----+-----+-----
Model |    18070.9007         5    3614.18013          F( 5, 4970) =     92.50
Residual |   194192.723       4970    39.0729825          Prob > F      =    0.0000
-----+-----+-----+-----+-----
Total |   212263.624       4975    42.666055          R-squared     =    0.0851
                                           Adj R-squared =    0.0842
                                           Root MSE     =    6.2508

-----+-----
dist1sum |           Coef.      Std. Err.      t    P>|t|     [95% Conf. Interval]
-----+-----+-----+-----+-----
b1      q0rec |   -1.735207       .4785371     -3.63  0.000     -2.673351   -.7970636
b2(I1) _Iskt3_1 |    1.829237       .3463889     5.28  0.000     1.150162    2.508312
b3(I2) _Iskt3_2 |   -0.5728095     .4673413     -1.23  0.220     -1.489005    .3433857
b4      _Iq0Xsk1 |  -2.383876       .5260111     -4.53  0.000     -3.41509    -1.352662
b5      _Iq0Xsk2 |  -0.3689434     .6391081     -0.58  0.564     -1.621877    .8839906
      _cons |   17.37157       .312152     55.65  0.000     16.75962    17.98353
-----+-----

```

estimates store m2

lrtest mo1 mo2

```

Likelihood-ratio test          LR chi2(2) =      32.23
(Assumption: mo1 nested in mo2)  Prob > chi2 =    0.0000

```

In this very case we do not test one single parameter, but rather compare two models where the saturated model has two more parameters (b4 and b5). Therefore the likelihood ratio test is the adequate way to decide between the restricted and the saturate model. Again, we should have a closer look at how each cell can be represented by an additive linear combination of the parameters. This will help decide which parameter (or combination of parameters) should be tested for which hypotheses.

$$\hat{y} = \text{cons} + b1 * q0rec + b2 * I1 + b3 * I2 + b4 * (q0rec * I1) + b5 * (q0rec * I2) + \text{_Iq0Xsk1} + \text{_Iq0Xsk2}$$

If you insert the corresponding values, you will get the following results:

$$y_{11} = \text{cons} + b1 (q0rec=0) + b2 I1 (I1=1) + b3 I2 (I2=0) + b4 (q0rec=0 * I1=1) + b5 (q0rec = 0 * I2 = 0) = \text{cons} + b1 * 0 + b2 * 1 + b3 * 0 + b4 * 0 + b5 * 0 = \text{cons} + b2$$

$$y_{11} = 17.371571 + 1.829237 = \mathbf{19.200808}$$

$$y_{21} = \text{cons} + b1 (q0rec=0) + b2 I1 (I1=0) + b3 I2 (I2=1) + b4 (q0rec=0 * I1=0) + b5 (q0rec = 0 * I2 = 1) = \text{cons} + b1 * 0 + b2 * 0 + b3 * 1 + b4 * 0 + b5 * 0 = \text{cons} + b3$$

$$y_{21} = 17.371571 + (-0.5728095) = \mathbf{16.798761}$$

$$y_{31} = \text{cons} + b1 (q0rec=0) + b2 I1 (I1=0) + b3 I2 (I2=0) + b4 (q0rec=0*I1=0) + b5 (q0rec = 0 * I2 = 0) = \text{cons} + b1*0 + b2*0 + b3* 0 + b4*0 + b5*0 =$$

cons

$$y_{31} = \mathbf{17.371571}$$

$$y_{12} = \text{cons} + b1 (q0rec=1) + b2 I1 (I1=1) + b3 I2 (I2=0) + b4 (q0rec=1*I1=1) + b5 (q0rec = 1 * I2 = 0) = \text{cons} + b1*1 + b2*1 + b3* 0 + b4*1 + b5*0 =$$

cons + b1 + b2 + b4

$$y_{12} = 17.371571 + (-1.735207) + 1.829237 + (-2.383876) = \mathbf{15.081725}$$

$$y_{22} = \text{cons} + b1 (q0rec=1) + b2 I1 (I1=0) + b3 I2 (I2=1) + b4 (q0rec=1*I1=0) + b5 (q0rec = 1 * I2 = 1) = \text{cons} + b1*1 + b2*0 + b3* 1 + b4*0 + b5*1 =$$

cons + b1 + b3 + b5

$$y_{22} = 17.371571 + (-1.735207) + (-0.5728095) + (-0.3689434) = \mathbf{14.694611}$$

$$y_{32} = \text{cons} + b1 (q0rec=1) + b2 I1 (I1=0) + b3 I2 (I2=0) + b4 (q0rec=1*I1=0) + b5 (q0rec = 1 * I2 = 0) = \text{cons} + b1*1 + b2*0 + b3* 0 + b4*0 + b5*0 =$$

cons + b1

$$y_{32} = 17.371571 + (-1.735207) = \mathbf{15.636364}$$

Table 16 shows in a more comprehensive way, how each of the 6 cells means (comp. Table 11) can be represented by an additive linear combination of the model parameters.

Table 16: Decomposition table

	q0rec	
skt3	0	1
1	cons + b2 <i>y₁₁</i>	cons + b1 + b2 + b4 <i>y₁₂</i>
2	cons + b3 <i>y₂₁</i>	cons + b1 + b3 + b5 <i>y₂₂</i>
3	cons <i>y₃₁</i>	cons + b1 <i>y₃₂</i>

The additive linear combinations (in bold face) provide the information on how to interpret the regression parameters. The non-saturated model simply assumes that both b_4 and b_5 are zero, which does not hold, as the `lrtest` clearly has shown. Now, what is called the simple slopes or conditional effect of the variable `q0rec` and both dummy variables `_Iskt3_1` and `_Iskt3_2` are not “main effects” any more. `q0rec` is the difference between the two groups indicated by this variable, but only for the respondents in category 3 (omitted

category) of $skt3$. To obtain the difference between the 2 case stories conditioned on category 2 of $skt3$ one has to add $b5$, because:

$$\Delta(Y_{21}, Y_{22}) = (\text{cons} + b3) - (\text{cons} + b1 + b3 + b5) = \mathbf{b1+b5}$$

Since $b5$ (-.3689434) is small and not significant we can assume the effect of $q0rec$ does not differ between category 2 and 3 of $skt3$; it remains to be $b1$.

$$\Delta(Y_{11}, Y_{12}) = (\text{cons} + b2) - (\text{cons} + b1 + b2 + b4) = \mathbf{b1+b4}$$

For the first category of $skt3$ this difference is $b4$ (-2.383876) and highly significant. We should conclude that the difference between $q0rec$ categories (vignettes) conditioned on category 1 of $skt3$ (labelled as mentally ill) is considerably larger than conditioned on category 3. Results are not surprising: Category 2 and 3 of $skt3$ are rather similar. There is not much impact on the effect of the case story. But for those who labelled the person described in the vignette as *mentally ill*, being presented with a schizophrenic episode or a major depression, is of great importance for their scoring on the social distance scale. The difference is more than 4 points, on a scale from 0 to 28 ($b1 + b4$). Compare the first row of Table 16.

2.4 Two predictors (one dichotomous and one continuous)

In this chapter we will demonstrate how a categorical variable, one continuous variable and their interaction may be implemented as predictors and how the parameters have to be interpreted. This chapter should be considered only as a short outline, further reading is necessary to become genuinely familiar with all the problems (Aiken & West, 1991). For this example we additionally plug in the age of the respondents as a predictor of social distance. Everything said about “simple slopes” in section 2.2 still holds. If an interaction is adopted as a predictor, which is simply the product of at least two predictors, the conditional effects are always the effects of one of the predictors, given the other predictor is zero. Therefore, the researcher has to know what “zero” actually means. In the case of a dummy coded categorical variable, this is quite simple and straightforward. For a continuous variable we have to take care that a value of zero is both in the range of the observable values and represents a value where the conditional effect for the categorical variable is really of interest. If we employ the age “as is”, both criteria are obviously violated. Observations with an age of zero definitely

are not a part of the sample **and** the population, and, of course, any test regarding this “group” of observations makes no sense at all. Usually, to solve this problem, continuous variables are centred at their (sample) mean, so the conditional effect is the effect at the mean of the continuous variable. We first estimate a model without and then with an interaction between agec (age centred at its sample mean) and q0rec, which can be found in Table 17 and Table 18 respectively.

Table 17: `xi3:regress dist1sum q0rec agec (age “mean centred”)`

Source	SS	df	MS			
Model	17572.1465	2	8786.07327	Number of obs =	4938	
Residual	192896.307	4935	39.0873976	F(2, 4935) =	224.78	
				Prob > F =	0.0000	
				R-squared =	0.0835	
				Adj R-squared =	0.0831	
				Root MSE =	6.252	
Total	210468.454	4937	42.6308393			

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
q0rec	-3.551218	.1779552	-19.96	0.000	-3.90009	-3.202347
agec	.0376527	.0051973	7.24	0.000	.0274638	.0478416
_cons	18.5861	.126621	146.79	0.000	18.33787	18.83434

Estimates store q0recage1

Table 18: `xi3:regress dist1sum q0rec*agec (with interaction)`

Source	SS	df	MS			
Model	17863.9569	3	5954.65228	Number of obs =	4938	
Residual	192604.497	4934	39.0361769	F(3, 4934) =	152.54	
				Prob > F =	0.0000	
				R-squared =	0.0849	
				Adj R-squared =	0.0843	
				Root MSE =	6.2479	
Total	210468.454	4937	42.6308393			

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
q0rec	-3.550065	.177839	-19.96	0.000	-3.898708	-3.201421
agec	.0231622	.0074206	3.12	0.002	.0086146	.0377098
_Iq0Xag	.0284069	.0103898	2.73	0.006	.0080383	.0487756
_cons	18.58454	.1265393	146.87	0.000	18.33646	18.83261

Estimates store q0recage2

To facilitate the comparison between the two hierarchically nested models, we employ another feature of the `estimates` command. Since the names of the models only differ with respect to the last character (1 or 2), we can shorten the command by using the wildcard “*”:

Table 19: `estimates table q0recage*,star stats(N r2 ll) b(%9.2f)`

Variable	q0recage1	q0recage2
q0rec	-3.55***	-3.55***
agec	0.04***	0.02**
_Iq0Xag		0.03**
_cons	18.59***	18.58***
N	4938.00	4938.00
r2	0.08	0.08
ll	-16056.08	-16052.34

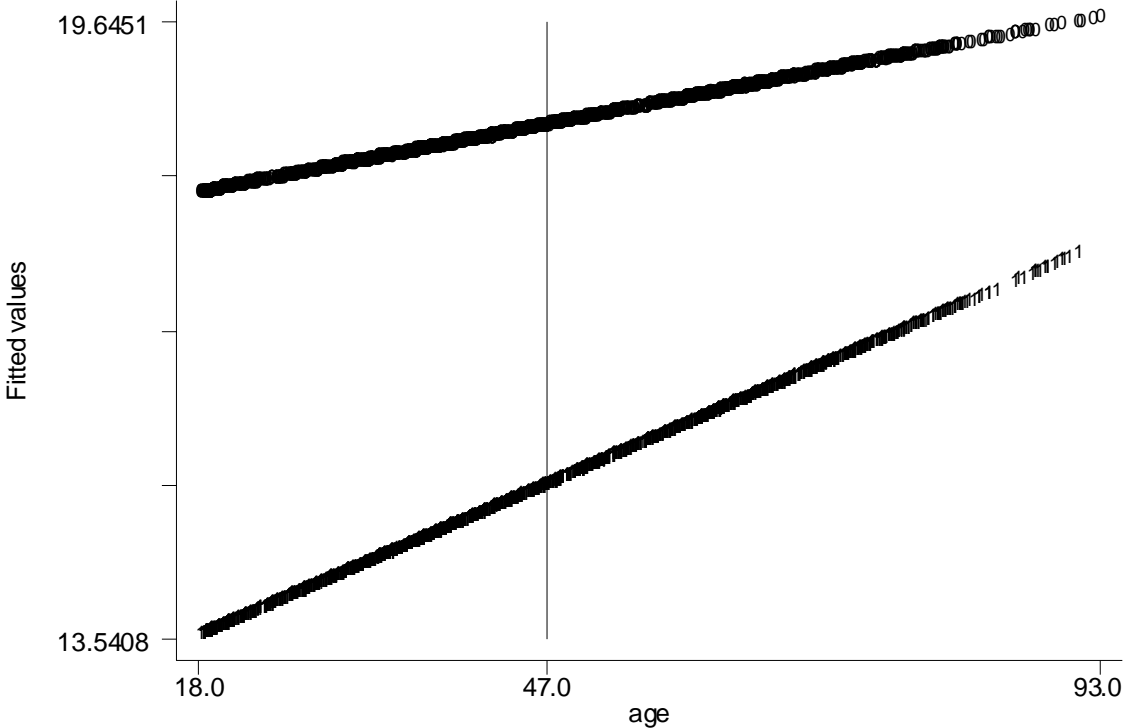
Legend: * p<0.05; ** p<0.01; *** p<0.001

We plot the predicted values of the second model against age and adopt the codes of `q0rec` for a plotting symbol. As said before, we employ the mean centred age (`agec`) as predictor for reasons presented above. The conditional effect of `q0rec` exactly represents and tests the difference in social distance for this value of age. Two important facts should not be overlooked:

1. The mean of age should be the mean age for only those observations which are used to estimate the regression.
2. The test of the conditional effect (here `q0rec`) is a test at the mean age of the *sample*. Of course, this value may differ from the mean of the population, which is not taken into account. However, for the purpose of this memo this might be of minor importance.

To get the predicted values for the saturated model, we type `predict distage2 if e(sample)` and plot the age against these variable.

Figure 1: Scatterplot of predicted values against the non-centred age (mean age 47)
`gr7 distage2 age,s([q0rec]) xlab(18,47,93) xline(47)`



We see that although the R^2 does not change at all, the likelihood test (and the interaction term) turned out to be significant. Here we are confronted with a very important problem of how to interpret model results: The difference between statistical and practical significance. Since the two lines presenting the linear fit for each of the two groups ($q0rec(0, 1)$), the conditional “slope” changes, if the value zero for the continuous variable changes its meaning. Of course, we could have centred the age variable for other values: for instance at 18 (youngest) and 90 (oldest), and it would be possible to centre age at 200, which will yield an out of range estimation for the conditional effect $q0rec: 0.79$, but this is not significant. If only our respondents would have reached an age of 200 years, the two case stories would produce no more difference at least with respect to social distance.

Of course one can get both the conditional slopes of $q0rec$ and their standard errors from the regression with a non-centred age. First we have a look at Table 20 below:

Table 20: Model age (without any centring) xi3:regress dist1sum q0rec*age

Source	SS	df	MS			
Model	17863.9569	3	5954.65229	Number of obs =	4938	
Residual	192604.497	4934	39.0361769	F(3, 4934) =	152.54	
Total	210468.454	4937	42.6308393	Prob > F =	0.0000	
				R-squared =	0.0849	
				Adj R-squared =	0.0843	
				Root MSE =	6.2479	

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
q0rec	-4.89123	.5213757	-9.38	0.000	-5.913359	-3.869102
age	.0231622	.0074206	3.12	0.002	.0086146	.0377098
_Iq0Xag	.0284069	.0103898	2.73	0.006	.0080383	.0487756
_cons	17.49099	.3717408	47.05	0.000	16.76221	18.21977

We see that only the conditional slope for `q0rec` changes: as expected, it is then even bigger in absolute value for the 18-centred age, but, as said before, this value is formally correct but meaningless. To get the “simple slope” for the 18 year old respondents, we briefly reconsider the linear equation already presented after Table 9, and rewrite them for the very problem outlined in this section.

$$\hat{y} = \text{cons} + (\mathbf{b1} + \mathbf{b3*age}) * \text{q0rec} + \text{b2*age} \text{ or:}$$

$$\hat{y} = \text{cons} + (\mathbf{b2} + \mathbf{b3*q0rec}) * \text{age} + \text{b1*q0rec}$$

The “simple slopes” are again printed in bold face, and their values depend on the value of age or `q0rec`. Here we we will only evaluate the first equation ($\mathbf{b1} + \mathbf{b3*age}$) because the second slope results in only two values : it is $\mathbf{b2}$ if `q0rec` is zero or $\mathbf{b2+b3}$ if `q0rec` is one. To estimate the coefficient for `q0rec` if the age is 18, the mean age, or 90, we compute using the `display` command:

```
display (-4.89123 + (.0284069*18)) = -4.379906
display (-4.89123 + (.0284069*47.17427)) = -3.551155
display (-4.89123 + (.0284069*90)) = -2.334609
```

Unfortunately, we do not currently know the standard errors of the simple slopes, but the equations above tell us that each of the coefficients is a weighted linear combination of coefficients for which the Variance/Covariance matrix is available from the analysis presented earlier (Table 20). Therefore the standard error of the simple slope is the square root of a weighted linear combination. We first store the variance/covariance matrix immediately after the regression analysis in a matrix arbitrarily called `ageV` by means of the command `matrix ageV = e(V)`.

Table 21: matrix list ageV (only subdiagonal displayed)

```

symmetric ageV[4,4]
      q0rec      age      _Iq0Xag      _cons
q0rec  .27183263
age    .0025938  .00005506
_Iq0Xag -.00509213 -.00005506  .00010795
_cons  -.13819122  -.0025938  .0025938  .13819122

```

If we want to estimate the variance of the linear combination for age 18, we first have to estimate the following combination of variances and covariances, which can be defined by looking at the square of the weighted linear combination of parameters:

$$(b1 + b3*18)^2 = (b1 + b3*18)^2$$

To obtain the combination of parameters necessary to estimate the variance of the simple slope we compute:

Equation 1:

$$s_{11} + s_{31}^2*18 + s_{31}*18 + s_{33}*18*18 = s_{11} + 2*s_{31}*18 + s_{33}*18^2$$

The square root of this expression then is the standard error of the respective coefficient. If we employ the display command and the row and column indices of the matrix age, we simply have to type:

```
display sqrt(ageV[1,1]+(2*ageV[3,1]*18) +(ageV[3,3]*(18*18)))
```

to obtain the expected standard error **.35141301**. If we would have estimated the model with age centred at 18, we would have obtained:

Table 22: Model age centred at 18

Source	SS	df	MS	Number of obs = 4938		
Model	17863.9569	3	5954.65229	F(3, 4934)	=	152.54
Residual	192604.497	4934	39.0361769	Prob > F	=	0.0000
Total	210468.454	4937	42.6308393	R-squared	=	0.0849
				Adj R-squared	=	0.0843
				Root MSE	=	6.2479

distlsum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
q0rec	-4.379906	.351413	-12.46	0.000	-5.068831	-3.69098
age18	.0231622	.0074206	3.12	0.002	.0086146	.0377098
_Iq0Xag	.0284069	.0103898	2.73	0.006	.0080383	.0487756
_cons	17.90791	.2503108	71.54	0.000	17.41719	18.39863

Estimating the model several times, with differently centred continuous predictors is certainly the simpler way to the desired results. However it is worth knowing, how these coefficients and standard errors are related. This holds for each possible (and impossible) value of the continuous predictor. If instead of 18, 90 is plugged into the equation, we will obtain the standard error of q_{0rec} for the 90 year old respondents. As expected the effects for the differently centred age variables and the respective interaction term remains invariant, but the conditional effect of q_{0rec} changes from -4.3 to -2.3 . The difference in social distance attributed to the case story becomes smaller if only the age of the respondents increases (Figure 1). We also see, that this difference remains significant for all observed values of age.

**Table 23: estimates table age18 agec age90,star stats(N r2)
b(%9.2f)**

Variable	age18	agec	age90
q0rec	-4.38***	-3.55***	-2.33***
age18	0.02**		
_Iq0Xag	0.03**	0.03**	0.03**
agec		0.02**	
age90			0.02**
_cons	17.91***	18.58***	19.58***
N	4938.00	4938.00	4938.00
r2	0.08	0.08	0.08

legend: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$

The more general algorithm to obtain the variances of linear combination of parameters needs a selection (row) vector w and the var/cov matrix already introduced above, which is called $ageV$ in the following. The vector w selects and weights the coefficients from the matrix $ageV$. The selection vector has as many as elements as the $ageV$ has rows (or columns). Since we need the square of combination we need to premultiply the matrix $ageV$ with w , and to postmultiply with w' , and we get:

Equation 2: $s_{comb} = w * ageV * w'$ (compare, for instance, Aiken & West 1991:24pp)

The square root of s_{comb} is the standard error of the coefficient. How must the weight vector w be defined? For the example above we need two parameters: b_1 represented by q_{0rec} , and b_3 represented by $_{Iq0Xag}$ in Table 21. Since we want to estimate the variance for an age of 18, the 2nd parameter need to be weighted by 18 (comp. Equation 1). Therefore, the row vector w has to be defined $w = [1 \ 0 \ 18 \ 0]$. The 2nd scalar is zero, since we do not need

the variance or covariance of any term with age: s_{22} . Since the constant also is not of interest, it must be weighted by zero either.

Equation 3:

$$[1 \ 0 \ 18 \ 0] \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 18 \\ 0 \end{pmatrix}$$

With Equation 3 (which is nothing but Equation 2 written out in full) the linear combination of variances can be obtained. Let us first look on the row vector resulting from $w \cdot \text{age}V$:

$$(s_{11}+s_{31} \cdot 18) \quad (s_{12}+s_{32} \cdot 18) \quad (s_{13}+s_{33} \cdot 18) \quad (s_{14}+s_{34} \cdot 18)$$

Now this 4-column vector is again multiplied by w' , which result in the expected scalar:

$$(s_{11}+s_{31} \cdot 18) + (s_{13}+s_{33} \cdot 18) \cdot 18 \text{ since } s_{13} = s_{31}$$

We obtain the same as in the second row of Equation 1:

$$s_{11} + 2 \cdot s_{31} \cdot 18 + s_{33} \cdot 18^2 \quad \text{q.e.d}$$

Study suggestions 2: The reader is invited to try out for interactions between only categorical variables.

Though we will come back to this problem below in section 3.3.1., there is no harm in trying, because the algorithm just presented holds for any kind of weighting, which only depends on the coding of the predictors. We elaborated on this topic for the combination of a categorical and a continuous variable only, because it demonstrates, that there is basically no difference between those types of variables, as long as we know what the numbers used for the coding of a predictor actually mean.

3 Deviation/Effect coding

In some instances we do not want to compare the mean of particular groups with a specified reference group but rather the mean of a subgroup with what is called the *grand mean*. This figure is the mean of all group means, and not the overall mean of the sample observed. Effect

coding then enables you to compare the mean of the dependent variable for a given level of a predictor to the mean of the dependent variable for all the levels of the variable. We will focus solely on non-orthogonal designs in non-experimental research.

3.1 One dichotomous predictor

First we will present a simple example with one dichotomous predictor only. We again employ the dichotomous predictor `sktd` (problem definition). To estimate and test the deviation from the *grand mean*, we need a new predictor variable now coded -1 and +1 for either of the 2 categories. We arbitrarily chose +1 for a new variable: `sktdeffect` if `sktd` indicates no definition of mental illness (code 1) and -1 for the other category. The regression analysis now looks like:

Table 24: regress dist1sum sktdeffect

Source	SS	df	MS			
Model	2049.24049	1	2049.24049	Number of obs =	4976	
Residual	210214.383	4974	42.2626424	F(1, 4974) =	48.49	
Total	212263.624	4975	42.666055	Prob > F =	0.0000	
				R-squared =	0.0097	
				Adj R-squared =	0.0095	
				Root MSE =	6.501	

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
sktdeffect	-.677629	.0973137	-6.96	0.000	-.8684067	-.4868513
_cons	16.57579	.0973137	170.33	0.000	16.38502	16.76657

First, let us calculate the grand mean:

$$(17.253423+15.898165)/2 = \mathbf{16.575794}$$
 which is now – as expected – the constant of the regression. Note, that this is **NOT** the total mean (e.g. the mean of all the 4976 observations). The means of the 2 cells can be estimated straightforward by:

$$y_1 = c + b_1(-1) = \mathbf{16.575794} + 0.677629 = ?$$

$$y_2 = c + b_1(1) = 16.575794 - 0.677629 = ? \text{ compare Table 3.}$$

Alternatively, we obtain the deviations from the *grand mean*:

$$(17.253423+15.898165)/2 - 17.253423 = \mathbf{-.677629}$$

$$(17.253423+15.898165)/2 - 15.898165 = \mathbf{.677629}$$

Of course, the effect is exactly half of the effect obtained from ordinary dummy coding (compare Table 2), but the test is different: the test for b_1 tells us that the deviation from the grand mean is significant.

3.2 Two dichotomous predictors (both coded as deviation from the grand mean)

In case of two dichotomous predictors, we can estimate the 4 cell means either by regarding only the marginal distribution or adopting a fully saturated model as before, with exactly 4 parameters for the 4 cells. The actual means are tabulated in Table 5 and the four cells are now defined by the following linear combination of regression parameters:

$$\begin{aligned} \hat{Y}_{11} &= c + (-1)b_1 + (-1)b_2 \\ \hat{Y}_{12} &= c + (1)b_1 + (-1)b_2 \\ \hat{Y}_{21} &= c + (-1)b_1 + (1)b_2 \\ \hat{Y}_{22} &= c + (1)b_1 + (1)b_2 \end{aligned}$$

To make things simple, we can use the `e` option of the pre-command `xi3`. Results of the analysis are presented in Table 25 below.

Table 25: `xi3:regress dist1sum e.sktd e.q0`

```
e.sktd          _Isktd_1-2          (naturally coded; _Isktd_1 omitted)
e.q0            _Iq0_1-2          (naturally coded; _Iq0_1 omitted)
```

Source	SS	df	MS	Number of obs = 4976		
Model	16744.0457	2	8372.02287	F(2, 4973)	=	212.94
Residual	195519.578	4973	39.3162232	Prob > F	=	0.0000
-----				R-squared	=	0.0789
-----				Adj R-squared	=	0.0785
Total	212263.624	4975	42.666055	Root MSE	=	6.2703

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_Isktd_2	-.507479	.0942719	-5.38	0.000	-.6922935	-.3226644
_Iq0_2	-1.726141	.0892854	-19.33	0.000	-1.90118	-1.551103
_cons	16.65194	.0939428	177.26	0.000	16.46777	16.83611

```
predict dp if e(sample)
estimates store distsume
```

Of course, sum of squares, mean squares and all the other characteristics of the model do not depend on the coding scheme; this also holds for the predicted values which are given below

(Table 26). They are exactly the same as those provided by dummy coding (compare Table 7). We should expect that the mean of the predicted means of the 4 cells should be represented by the constant of the regression analysis above.

$$\text{display } (18.885563+15.433281+17.870605+14.418323)/4 = \mathbf{16.651943}$$

This can be easily verified by calculating the mean employing the definition for each cell:

$$\begin{aligned} & ((c + (-1)b1 + (-1)b2) + \\ & (c + (1)b1 + (-1)b2) + \\ & (c + (-1)b1 + (1)b2) + \\ & (c + (1)b1 + (1)b2)) / 4 \end{aligned}$$

Since all the terms except c cancel out, we obtain: $4c/4$ q.e.d.

Table 26: `tab sktd q0, sum(dp)` dp = predicted y from Table 25.

sktd	Schizoph	Depressi	Total
1	18.885563	15.433281	17.253422
	.	0	1.7238418
	1733	1554	3287
2	17.870605	14.418323	15.898164
	0	.	1.7089851
	724	965	1689
Total	18.586487	15.044462	16.793408
	.46280669	.49350943	1.8345671
	2457	2519	4976

Without an interaction effect, the conditional differences between columns are necessarily all the same for each row, and this is (statistically) true if the effect for a product of X_1 and X_2 turns out to be zero.

$$y_{11} - y_{12} \mid x_2 == -1 = \mathbf{-2b1}$$

$$y_{21} - y_{22} \mid x_2 == +1 = \mathbf{-2b1}$$

Since the deviation from the *grand mean* is modelled, the actual difference between the categories is twice the respective parameter. This holds both for the difference with respect to rows and columns. With interaction the equation becomes just the same as for dummy coding, even though the design matrix is different.

If both X1 and X2 are coded equally (-1 or +1) the design for the interaction is necessarily 1. If X2 and X2 are coded differently the numerical value for the product is consistently -1. The estimated means for each of the 4 cells represent the actual means for the 2x2 table.

$$\begin{aligned} \hat{Y}_{11} &= c + (-1)b_1 + (-1)b_2 + (1)b_3 \\ \hat{Y}_{12} &= c + (1)b_1 + (-1)b_2 + (-1)b_3 \\ \hat{Y}_{21} &= c + (-1)b_1 + (1)b_2 + (-1)b_3 \\ \hat{Y}_{22} &= c + (1)b_1 + (1)b_2 + (1)b_3 \end{aligned}$$

Now the difference of means for one of the predictors conditioned on the categories of the other predictor can not be represented any more by one single parameter, as we have just seen for dummy coding.

$$\begin{aligned} y_{11} - y_{12} \mid X_2 == -1 &= -2b_1 + 2b_3 \\ y_{21} - y_{22} \mid X_2 == +1 &= -2b_1 - 2b_3 \end{aligned}$$

One should not forget that both b1 and b2 are **NOT** the same as before, where we estimated the differences under the assumption that they are equal for both categories of the other variable. The numerical results rather look like:

Table 27. xi3:regress dist1sum e.sktd*e.q0rec

```

e.sktd          _Isktd_1-2          (naturally coded; _Isktd_1 omitted)
e.q0rec         _Iq0rec_0-1        (naturally coded; _Iq0rec_0 omitted)

-----+-----
Source |           SS          df           MS          Number of obs =    4976
-----+-----
Model  |    17829.863           3    5943.28765      F( 3, 4972) =    151.98
Residual |  194433.761       4972    39.1057443      Prob > F      =    0.0000
-----+-----
Total  |  212263.624       4975    42.666055      R-squared     =    0.0840
                                           Adj R-squared =    0.0834
                                           Root MSE    =    6.2535

-----+-----
dist1sum |           Coef.      Std. Err.      t    P>|t|     [95% Conf. Interval]
-----+-----
_Isktd_2 |   -0.5455136      0.0942959    -5.79  0.000    -0.7303752   -0.360652
_Iq0rec_1 |  -1.562662      0.0942959   -16.57  0.000   -1.747524   -1.377801
_Isk2Xq01 |   0.4968793      0.0942959    5.27  0.000    0.3120177    0.6817408
_conds   |   16.59575      0.0942959   176.00  0.000   16.41089    16.78061
-----+-----

```

estimates store **distsumeint**

A likelihood test shows that the interaction should be taken into account. Again all the result are the same as already reported in section 2.2 (also compare Table 8) .

Table 28: `lrtest dist1sume dist1sumeint`

```
Likelihood-ratio test                                LR chi2(1) =      27.71
(Assumption: dist1sume nested in dist1sumeint)       Prob > chi2 =      0.0000
```

This likelihood test necessarily yields the same results as with dummy coding of both predictors (Table 10). It should be emphasized that the two different coding schemes represent the same model, as far as prediction and explained variance are concerned. The meaning of the parameters (point of reference) is, of course, different. Statistical inference for the one and only one interaction parameter remains invariant, although the numerical values for these terms are necessarily different.

3.3 Predictors with more than 2 categories

Things become more general if we want to employ a predictor with more than 2 categories. We now take the same concept (`skt`) as before, but with 3 categories. The second category of `sktd` (no definition as mental illness) can be divided into 2 categories, so we obtain:

1. mental illness
2. illness
3. neither 1 nor 2

Let's first look at the table of means and the *grand mean*, which will become of interest in the following:

Table 29: `tab skt3, sum(dist1sum)`

skt3	Mean	Std. Dev.	Freq.
1	17.253423	6.4038822	3287
2	15.380424	6.7361216	991
3	16.633238	6.5488024	698
Total	16.793408	6.5319258	4976

To obtain the *Grand Mean* (the mean of the means) you have to type:

```
display (17.253423 + 15.380424 + 16.633238) / 3 = 16.42236
```

3.3.1 Manual Coding

As you see in the example below (Table 30), the regression scheme for effect coding is accomplished by assigning 1 to level 2 for the first comparison (because level 2 is the level to be compared to all), level 1 to level 3 for the second comparison (because level 3 is to be compared to all). Note that a -1 is assigned to level 1 for all three comparisons (because it is the level that is never compared to the other levels) and all other values are assigned a 0. This regression coding scheme yields the comparisons described above. It also means that the effect of the omitted category is simply the negative sum of the other effects. We call the new indicator variables X2 and X3, since the category 1 is adopted for to be the omitted category.

Table 30: Contrast Matrix for 3 categories (first category as reference category)

	New variable 1 (X2)	New variable 2 (X3)
skt3	Level 2 v. Mean	Level 3 v. Mean
1	-1	-1
2	1	0
3	0	1

Based on this scheme, we illustrate how to create X2 and X3 and enter these new variables into the regression model.

```
generate x2 = -1 if skt3==1
replace x2 = 1 if skt3==2
replace x2 = 0 if skt3==3

generate x3 = -1 if skt3==1
replace x3 = 1 if skt3==3
replace x3 = 0 if skt3==2
```

Table 31. regress dist1sum x2 x3

Source	SS	df	MS			
Model	2692.03569	2	1346.01785	Number of obs =	4976	
Residual	209571.588	4973	42.1418838	F(2, 4973) =	31.94	
Total	212263.624	4975	42.666055	Prob > F =	0.0000	
				R-squared =	0.0127	
				Adj R-squared =	0.0123	
				Root MSE =	6.4917	

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x2	-1.041938	.1644163	-6.34	0.000	-1.364266	-.7196092
x3	.2108764	.181612	1.16	0.246	-.1451631	.566916
_cons	16.42236	.1133924	144.83	0.000	16.20006	16.64466

To calculate the means of the dependent variable shown in Table 29, you have to use the equation for the linear regression model and the obtained coefficients from the last STATA output.

The equation for this model now appears as follows:

$$\hat{y} = \text{cons} + b_1(X_2) + b_2(X_3)$$

As before, the constant is the *grand mean*. Do not confuse “reference category” with “omitted category”!!! The mean for the second category is:

$$\hat{y}_2 = 16.42 + (-1.04)(1) + 0.21(0)$$

$$\hat{y}_2 = 15.38$$

and for the third category :

$$\hat{y}_3 = 16.42 + (-1.04)(0) + 0.21(1)$$

$$\hat{y}_3 = 16.63$$

\hat{y}_2 represents the mean of `dist1sum` for people with `skt3=2`. By simply typing: `display 15.38-16.42` we can confirm that this result is a value identical to the regression coefficient of `X2`.

\hat{y}_3 represents the mean of `dist1sum` for people with `skt3=3`. The coefficient for `x3` is the mean for level 3 of `skt3` minus the grand mean, i.e., **16.63 - 16.42 = 0.21**.

You can make your life easier, using the `xi3` pre-command and taking the option `e`, writing `e.skt3` as shown below. With the command `refgroup` you can set the omitted category for the independent variable.

refgroup skt3 1

Table 32 **xi3:reg dist1sum e.skt3**

e.skt3		_Iskt3_1-3		(naturally coded; _Iskt3_1 omitted)		
Source	SS	df	MS	Number of obs = 4976		
Model	2692.03569	2	1346.01785	F(2, 4973) = 31.94		
Residual	209571.588	4973	42.1418838	Prob > F = 0.0000		
Total	212263.624	4975	42.666055	R-squared = 0.0127		
				Adj R-squared = 0.0123		
				Root MSE = 6.4917		
dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_Iskt3_2	-1.041938	.1644163	-6.34	0.000	-1.364266	-.7196092
_Iskt3_3	.2108764	.181612	1.16	0.246	-.1451631	.566916
_cons	16.42236	.1133924	144.83	0.000	16.20006	16.64466

As expected, the regression coefficients in Table 32 are identical to those in the example with the manually generated codes (Table 31). Of course, one is often interested in additionally knowing to what extent the omitted category deviates from the grand mean. We know from before that the effect is the negative sum of the other effects, so we can either calculate this “by hand”, or type:

```
display _b[_cons] - _b[ _Iskt3_2] - _b[ _Iskt3_3] which will result
in: 17.253423. If we compute the negative sum of the two parameters by the command
display - _b[ _Iskt3_2] - _b[ _Iskt3_3] we obtain 0.83106117. The
sum of the constant and this negative sum results in :
```

```
display _b[_cons] + _b[ _Iskt3_1] = 17.253423 (comp. Table 29).
```

Unfortunately this computation does not provide any statistical test. To obtain the respective t-value, we simply have to change the “omitted category” (using the `refgroup` command) and estimate the coefficients again. If you have no access to the `refgroup` command, you have to type: `char skt3[omit] x` where `x` stands for the category you want to become the omitted one. Verify that everything else remains the same and try without using the nice features of `refgroup` and `char` by creating the necessary variables “by hand”.

We also can estimate and test the coefficient for the “omitted category” by means of the command `lincom`, which computes and tests point estimates for linear combination of parameters after estimation.

Table 33: `lincom -(_Iskt3_2 + _Iskt3_3)`

```
( 1) - _Iskt3_2 - _Iskt3_3 = 0
```

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
(1)	.8310612	.1308871	6.35	0.000	.5744647 1.087658

Although we got both the parameter and its standard error by means of the `lincom` command, we will show how to compute the standard error from the Var/Covar matrix of parameters. We employ the procedure already presented in section 2.4. For the example to be found in Table 32 the command `matrix list e(V)` will yield:

Table 34: symmetric e(V)[3,3]

	<code>_Iskt3_2</code>	<code>_Iskt3_3</code>	<code>_cons</code>
<code>_Iskt3_2</code>	.02703271		
<code>_Iskt3_3</code>	-.02144209	.03298291	
<code>_cons</code>	.00131703	.00726722	.01285784

The matrix in Table 34 provides all the information necessary to estimate the standard error of a linear combination of the two parameters. We again have to define a weight vector w , to select the adequate variances from the variance/covariance matrix of the parameters. We have to weight down the constant; therefore this row vector is (using STATA mnemonics) defined as:

```
matrix input w =(1 1 0)
```

To estimate the standard error of `_Iskt3_2 + _Iskt3_3`, we have to compute (comp. Equation 2 on page 21)

$S_b = \sqrt{w * e(V) * w'}$ = **0.13088709**. The actual command looks a bit different, because we need 2 commands. The first one defines the matrix operation which results in a scalar, the second one computes the square root of the one and only element of the matrix x .

```
matrix x=w*e(V)*w'  
display sqrt(x[1,1])
```

This is exactly the standard error of the test on linear combinations we obtained from the analysis presented in Table 33 .

Study suggestions 3: The reader is encouraged to solve the following matrix equation and to compare with results presented earlier.

$$[1 \ 1 \ 0] \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

3.3.2 A three-categorical and a dichotomous independent variable (Effect coding and dummy coding combined)

In a 2-dimensional table it is of interest to estimate deviations from a *grand mean* conditioning on another (perhaps dichotomous) characteristic of the sample. This means that we have a combination of effect- and dummy coding. To keep things simple we again adopt `skt3` using deviation/effect coding: `e.skt3` and the dichotomous variable `q0rec`. The reference category for `skt3` as well as for `q0rec` will be set to 1 and 0 respectively. The variables `_Iskt3_2` `_Iskt3_3` represent the deviation of category 2 or 3 from the grand mean respectively, similar to the example before. The effect for the omitted category is the negative sum of the other parameters. The effect `q0rec` estimates the mean difference between the two case stories (with “schizophrenia” as the reference category. The predicted values of the dependent variable can be obtained by the command: `predict dist1sume if e(sample)`. The means of `dist1sume` for each of the 6 cells can be found in Table 11.

Table 35: `xi3:regress dist1sume q0rec e.skt3`

```
e.skt3          _Iskt3_1-3          (naturally coded; _Iskt3_1 omitted)
```

Source	SS	df	MS			
Model	16809.1834	3	5603.06113	Number of obs =	4976	
Residual	195454.44	4972	39.3110298	F(3, 4972) =	142.53	
Total	212263.624	4975	42.666055	Prob > F =	0.0000	
				R-squared =	0.0792	
				Adj R-squared =	0.0786	
				Root MSE =	6.2699	

	dist1sume	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
b1	q0rec	-3.419012	.1804199	-18.95	0.000	-3.772715	-3.06531
b2	_Iskt3_2	-.529245	.1610862	-3.29	0.001	-.845045	-.213445
b3	_Iskt3_3	-.1262784	.1763061	-0.72	0.474	-.4719161	.2193594
	_cons	18.21431	.1446921	125.88	0.000	17.93065	18.49797

```
predict dist1sume if e(sample)
```

Of course, these values are not the actual means of the sample, since the linear model is not saturated. We have 6 cells estimated by only 4 parameters. Assuming that this restricted model is sufficient, we have to answer 3 main questions.

1. What is actually portrayed by the constant `_cons` ?
2. What deviations are actually portrayed by the effects `_Iskt3_2` `_Iskt3_3` ?
3. What difference is estimated by `q0rec` ?

The answer is straightforward, as for dummy coding. All we need to do is looking on how the 6 cells are assembled by the 4 parameters of the model:

$$\begin{aligned}
 \hat{y}_{11} &= c + (-1)\text{skt3_2} + (-1)\text{skt3_3} + (0)\text{q0rec} \\
 \hat{y}_{21} &= c + (1)\text{skt3_2} + (0)\text{skt3_3} + (0)\text{q0rec} \\
 \hat{y}_{31} &= c + (0)\text{skt3_2} + (1)\text{skt3_3} + (0)\text{q0rec} \\
 \hat{y}_{12} &= c + (-1)\text{skt3_2} + (-1)\text{skt3_3} + (1)\text{q0rec} \\
 \hat{y}_{22} &= c + (1)\text{skt3_2} + (0)\text{skt3_3} + (1)\text{q0rec} \\
 \hat{y}_{32} &= c + (0)\text{skt3_2} + (1)\text{skt3_3} + (1)\text{q0rec}
 \end{aligned}$$

Table 36: `tab skt3 q0, sum(dist1sume)`

Means, Standard Deviations and Frequencies of Fitted values

Welcher Fragebogen?			
skt3	Schizoph	Depressi	Total
1	18.869835	15.450822	17.253423
	.	2.771e-06	1.7072295
	1733	1554	3287
2	17.685066	14.266053	15.380424
	0	.	1.6033776
	323	668	991
3	18.088032	14.66902	16.633237
	.	.	1.6916361
	401	297	698
Total	18.586488	15.044462	16.793408
	.45175368	.52847	1.8381326
	2457	2519	4976

One can see immediately that the constant is the *grand mean* conditioned that `q0rec` is zero (e.g. the cases story of a schizophrenic episode was provided). The two effects for `skt3` (`_Iskt3_2_Iskt3_3`) now estimate the deviation from this mean, also on the condition that `q0rec` is zero.

There is only one parameter to map the difference(s) between the 2 columns of the 3 x 2 table (Table 36). Therefore the difference of 3.419 must necessarily hold for each row of the table concerning the predicted means. This also means that the category deviations are all equal for both values of `q0rec`. Adopting the parameter notation from Table 35 the decomposition is straightforward:

$$\begin{aligned}\hat{y}_{11} &= c + (-1)skt3_2 + (-1)skt3_3 + (0)q0rec + (0)_Iq0Xsk2 + (0)_Iq0Xsk3 \\ \hat{y}_{21} &= c + (1)skt3_2 + (0)skt3_3 + (0)q0rec + (0)_Iq0Xsk2 + (0)_Iq0Xsk3 \\ \hat{y}_{31} &= c + (0)skt3_2 + (1)skt3_3 + (0)q0rec + (0)_Iq0Xsk2 + (0)_Iq0Xsk3\end{aligned}$$

$$\begin{aligned}\hat{y}_{12} &= c + (-1)skt3_2 + (-1)skt3_3 + (1)q0rec + (0)_Iq0Xsk2 + (0)_Iq0Xsk3 \\ \hat{y}_{22} &= c + (1)skt3_2 + (0)skt3_3 + (1)q0rec + (1)_Iq0Xsk2 + (0)_Iq0Xsk3 \\ \hat{y}_{32} &= c + (0)skt3_2 + (1)skt3_3 + (1)q0rec + (0)_Iq0Xsk2 + (1)_Iq0Xsk3\end{aligned}$$

To calculate the grand mean we type the following:

$$\text{display } (19.20 + 16.80 + 17.37)/3 = 17.79$$

which is the grand mean for the left column ($q0rec == 0$) and the grand mean for the condition $q0rec == 1$ is

$$\text{display } (15.082 + 14.695 + 15.636)/3 = 15.137$$

The difference between the two grand means (one for each column) is -2.652814 which is the effect of $q0rec$. This is different from dummy coding (compare the previous chapter) where – in the case of interaction – the conditional parameter for $q0rec$ is valid for the omitted category of $skt3$ only. To get all the possible parameters, we recalculate the model with another reference category for $skt3$, and we obtain:

Table 39: xi3:reg dist1sum q0rec*e.skt3

e.skt3		_Iskt3_1-3		(naturally coded; _Iskt3_3 omitted)			
Source	SS	df	MS	Number of obs = 4976			
Model	18070.9007	5	3614.18013	F(5, 4970) = 92.50			
Residual	194192.723	4970	39.0729825	Prob > F = 0.0000			
-----				R-squared = 0.0851			
Total	212263.624	4975	42.666055	Adj R-squared = 0.0842			
-----				Root MSE = 6.2508			
dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]		
b1 q0rec	-2.652814	.2251294	-11.78	0.000	-3.094167	-2.211461	
b2 _Iskt3_1	1.410428	.1851707	7.62	0.000	1.047411	1.773444	
b3 _Iskt3_2	-.9916186	.2590284	-3.83	0.000	-1.499429	-.4838086	
b4 _Iq0Xsk1	-1.466269	.2580308	-5.68	0.000	-1.972124	-.9604152	
b5 _Iq0Xsk2	.548663	.332421	1.65	0.099	-.1030289	1.200355	
_cons	17.79038	.1636236	108.73	0.000	17.46961	18.11115	

estimates store dist1sumpint

The decompositions of each mean again provide the necessary information on interpreting these parameters. The “decomposition table” in Table 42 now looks a bit different. Parameters in italics represent the contribution of the interaction. Parameter notation is adopted from Table 39; cell means can be found in Table 11.

Table 42: **Decomposition table (compare Table 37)**

skt3		q0rec	
	1	cons + b2	cons + b1+ b2 + <i>b4</i>
	2	cons + b3	cons + b1+ b3 + <i>b5</i>
	3	cons + (b2+b3)*-1	cons + b1 + (b2+b3)*-1 + (<i>b4+b5</i>)*-1

3.3.3 An example from the ESEMED study (6 countries and *dsm_mde* / *mcs12* and age)

In this section we will show an example a bit more complicated than those provided above. In the ESEMED study (comp. 1.2) we have 6 countries which might be of interest to compare with respect to the outcome of the 2 SF12 scores : mental and physical health. For simplicity we will focus on the mental health score as a dependent variable only. For demonstrative purposes we will again present two models: one with two categorical predictors and one with a categorical and a continuous predictor. For a second discrete predictor we employ *dsm_mde* mapping lifetime depression as a dichotomy. The variable *dsm_mde* is already coded 0/1, so we have nothing to do. For a continuous predictor we adopt the age of our respondents called *age_main*. Centering will be discussed in the following.

3.3.3.1 Two categorical predictor (country and *dsm_mde*)

The two models are estimated by means of the following commands:

```

xi3:regress mcs12 e.countryn dsm_mde
estimates store mcs12                                and
xi3:regress mcs12 e.countryn*dsm_mde
estimates store mcs12int

```

We can tabulate and compare the results from Table 43 quite easily. We do not present the whole output but rather present both models by the `table` option of the `estimates` command. The first column shows results without `and`, the second column results for the saturated model. The likelihood ratio test tells that the interaction between the countries and the depression variable is statistically important. If no interaction is defined the differences between depression/no depression should be -6.9643 in each of the countries. Of course this is only true if the restricted model (without interaction) holds, which is not the case, as the `lrtest` clearly shows. We know that the difference between depression/no depression is not the same for each country. These differences are represented by the interaction effects. Since the row variable “country” is coded by deviations from the grand mean, the conditional effect called “`dsm_mde`” now represents the difference of the two grand means. This applies for with and without interaction (which is not true for dummy coding !!), although the exact figure must be different, because the estimated cell means for the restricted model can not be the same as for the saturated model - unless all the interaction effects would be zero.

Interaction effects show that only for Italy and Spain (4 and 6) is the difference in mental health QoL between depressive and non depressive people significantly larger than the average difference. The interaction effects do not test the difference in QoL for depression versus non-depression for a particular country compared to the omitted category, although the output tempts to interpret the table this way (Table 43). The output does not provide a parameter for the omitted category. If one wants to get the deviation from the grand mean, one simply should estimate the model a second time, and adopt another category to be “omitted”. From Table 45 we depict that for both the conditional effects and the interaction terms the parameter for the omitted country (Belgium) is the negative sum of the other parameters. In Table 43 and Table 45 the indicator variables are labelled with the initial characters of each country.

Table 43: `estimates table mcs12 mcs12int,stats(N ll r2) b(%10.3f) star`

Variable	mcs12	mcs12int
F _Icountryn_2	-0.728***	-0.816***
G _Icountryn_3	0.331**	0.343**
I _Icountryn_4	-0.865***	-0.735***
N _Icountryn_5	0.844***	0.735***
S _Icountryn_6	-0.509***	-0.346**
dsm_mde	-6.964***	-6.855***
F*D _Ico2Xds		0.469
G*D _Ico3Xds		0.167
I*D _Ico4Xds		-1.054**
N*D _Ico5Xds		0.647
S*D _Ico6Xds		-1.120***
_cons	54.839***	54.798***
N	21397.000	21397.000
ll	-73964.586	-73950.435
r2	0.094	0.095

legend: * p<0.05; ** p<0.01; *** p<0.001

Table 44: `lrtest mcs12 mcs12int`

Likelihood-ratio test LR chi2(5) = 28.30
 (Assumption: mm nested in mpint) Prob > chi2 = 0.0000

We see that statistics stay the same for both estimations (Table 45), but we now know, that also for Belgium (category 1 of `countryn`) the difference for depression deviates from the grand mean difference significantly. But here the difference is *smaller* than the grand mean, whereas for Italy and Spain the difference is, as said before, *bigger* than the grand mean, which is -6.8554576. The difference between the two categories of `dsm_mde` for Italy is -7.909951, which is -1.0545 bigger than the grand mean. This is exactly the effect for `_Ico4Xds` (comp. Table 45). However, the real differences resulting from the interaction effects are small, only for Italy and Spain are these terms significant, meaning the differences produced by `dsm_mde` are greatest for these countries. All the other effects never exceed the value of 1 on a scale from 9 to 72. Predicted values for both models are provided in Table 46. We also see from Table 45 that the conditional effect for `dsm_mde` does not change, even if the reference category for country is changed. This, of course, is one of the main advantages of effect coding as provided by the option `e` of `xi3`. Information contained in the first column is also provided in the second column of Table 43, so this table is a bit redundant.

Table 45: `xi3:regress mcs12 e.countryn*dsm_mde (Belgium or Spain omitted)`
`estimates table mcs12int mcs12intref1,star stats(N ll r2)`

Variable	Omitted 1	Omitted 6
B _Icountryn_1		.82012292***
F _Icountryn_2	-.81566565***	-.81566565***
G _Icountryn_3	.34257119**	.34257119**
I _Icountryn_4	-.73533133***	-.73533133***
N _Icountryn_5	.73455273***	.73455273***
S _Icountryn_6	-.34624986**	
dsm_mde	-6.8554576***	-6.8554576***
B*D _Ico1Xds		.8908707*
F*D _Ico2Xds	.4690798	.4690798
G*D _Ico3Xds	.16708308	.16708308
I*D _Ico4Xds	-1.0544945**	-1.0544945**
N*D _Ico5Xds	.64718754	.64718754
S*D _Ico6Xds	-1.1197266***	
_cons	54.79821***	54.79821***
N	21397	21397
ll	-73950.435	-73950.435
r2	.09538222	.09538222

legend: * p<0.05; ** p<0.01; *** p<0.001

Of course; one can also interpret this table of effects the other way around, saying that the deviations of the countries from the grand mean are not equal for a depressive and a non-depressive subpopulation. But this will certainly not be an interesting way to talk about the results, even though the statistical model for both interpretations is absolutely the same. There is no way to decide empirically which interpretation is a “correct” one, since this model does not distinguish between the predictors in a sense of a “causal” order. Predictors may be addressed to as “factors”; which means that the country “generates” differences regarding the effect of depression; or that depression “generates” differences with respect to the countries. However, this is not a question these models answer.

This holds, of course, for all the other examples in this textbook. Questions on causality can never be answered from the relation between variables, whether or not interactions between the predictors are part of the model. Attempts to solve this problem can be found quite frequently in the literature, but go far beyond the scope of this small booklet.

Table 46: `tab countryrn dsm_mde,sum(mcs12p)`

countryrn	dsm_mde		Total
	0	1	
1	55.766422	48.802135	54.734783
	.00001208	.	2.4744479
	2053	357	2410
2	54.110615	47.146324	52.567282
	.	.	2.8929641
	2248	640	2888
3	55.169487	48.205196	54.444981
	8.260e-06	.	2.1265082
	3178	369	3547
4	53.973915	47.009628	53.318749
	.0000159	0	2.033323
	4266	443	4709
5	55.682884	48.718594	54.300009
	9.050e-06	.	2.7787871
	1901	471	2372
6	54.329456	47.36517	53.485493
	.00001662	.	2.2729434
	4808	663	5471
Total	54.664553	47.760298	53.714924
	.68023851	.71105427	2.4745809
	18454	2943	21397

`tab countryrn dsm_mde,sum(mcs12intp)`

countryrn	dsm_mde		Total
	0	1	
1	55.618332	49.65374948	54.734781
	.	0	2.1192483
	2053	357	2410
2	53.982544	47.596169	52.567281
	0	.	2.6528983
	2248	640	2888
3	55.140781	48.452408	54.444979
	.	0	2.0422584
	3178	369	3547
4	54.062878	46.152927	53.318748
	.00001426	.	2.3094229
	4266	443	4709
5	55.532764	49.324493	54.300009
	8.010e-06	0	2.4771315
	1901	471	2372
6	54.451962	46.476776	53.485493
	.00001773	0	2.6028717
	4808	663	5471
Total	54.664552	47.760299	53.714923
	.60831955	1.2856862	2.4902594
	18454	2943	21397

3.3.3.2 One categorical and one continuous predictor

We already presented a combination of categorical and continuous predictors in section 2.4 but for the case of dummy coding. Centring of the continuous predictor in order to get a sound interpretation of the conditional effects for the other variable has been briefly addressed, but will become most important in this chapter. We will restrict ourselves to only one example, and again show how not only the parameters of the omitted category can easily be computed as linear combinations of the coefficients at hand, but also how the standard errors can be calculated from the variance/covariance matrix of these parameters, as was outlined the first time in section 3.3.1. This becomes important, since the employment of a continuous variable and interaction terms with other (perhaps categorical) predictors always results in conditional effects only for a particular value (zero) of this continuous predictor. However, it is possible, independent from the centring of the continuous variable, to estimate both the simple slopes and their standard errors for all possible and “impossible” (out of range) values of this variable.

First of all, we should have a look at two models without interaction, one of them with the “original” age (`age_main`) and the second one with mean centred age (`agemean`). Of course, only the constant will be different (Table 47). We show both analyses simultaneously below and will ask why the two constant must be different, and which point these two coefficients actually represent for the data.

Table 47: `estimates table age_main agemean,star stats(N ll r2)`

Variable	age_main	agemean
_Icountry_1	.91849696***	.91849696***
_Icountry_2	-1.2197568***	-1.2197568***
_Icountry_3	.63022879***	.63022879***
_Icountry_4	-.46775936***	-.46775936***
_Icountry_5	.47907764**	.47907764**
age_main	.0216037***	
agemean		.0216037***
_cons	52.775337***	53.808473***
N	21397	21397
ll	-74922.285	-74922.285
r2	.00935887	.00935887

legend: * p<0.05; ** p<0.01; *** p<0.001

We should expect the constant for a regression with `age_main` to be the *grand mean* for an observation with an age of zero. Unfortunately we can not obtain this (predicted) value simply by estimating the predicted values, since there are no respondents with an age of zero. Therefore we must estimate the predicted values “by hand”:

```
gen mcs1200=_b[_cons]*_cons + _b[_Icountry_1]*_Icountry_1 +
_b[_Icountry_2]*_Icountry_2 + _b[_Icountry_3]*_Icountry_3 +
_b[_Icountry_4]*_Icountry_4 + _b[_Icountry_5]*_Icountry_5 +
_b[age_main]*A if e(sample)
```

The A is a wildcard which represents a particular value of age, for which we want to estimate the dependent variable. We compute the predicted values twice, once for an age of zero and a second time for the mean age (47.82217). For both computations parameters from the left column of Table 47 are used. Tabulating these predictions with respect to the 6 countries, we obtain in Table 48:

Table 48: `tab country,sum(mcs120) for age mean and age zero`

Country	Mean	zero	Frequ.
Belgium	54.726971	53.693832	2410
France	52.588715	51.55558	2888
Germany	54.438702	53.405567	3547
Italy	53.340714	52.307579	4709
Netherland	54.287552	53.254414	2372
Spain	53.468185	52.435051	5471
Total	53.714923	52.681788	21397

```
display (54.726971 + 52.588715 + 54.438702 + 53.340714 +
54.287552 + 53.468185)/6 = 53.808473
```

```
display (53.693832 + 51.55558 + 53.405567 + 52.307579 +
53.254414 + 52.435051 )/6 = 52.775337
```

Compare these two grand means with the constants in Table 47 ! The difference between the two grand means is nothing but the change in the dependent variable from an age of zero to the mean age of the sample (47.82217 years). If we compute the value for the last term for the mean age, we obtain: `display 0.0216037*47.82217 = 1.033135` which must be `53.714923 - 52.681788 =`! Since there is no interaction effect, this value must hold for each country. But we will see immediately that this is not true. Estimating the model with the

necessary 5 interaction terms for two different omitted categories, we obtain the following results:

Table 49: estimates table belgium spain,star stats(N r2 l1) without centring of age

Variable	Belgium	Spain
B _Icountry_1		-.12261529
F _Icountry_2	-2.2414944***	-2.2414944***
G _Icountry_3	-1.3045075***	-1.3045075***
I _Icountry_4	2.1020507***	2.1020507***
N _Icountry_5	-.13278772	-.13278772
S _Icountry_6	1.6993543***	
age_main	.03385426***	.03385426***
B _Ico1Xag		.02160779*
F _Ico2Xag	.0221729**	.0221729**
G _Ico3Xag	.04023132***	.04023132***
I _Ico4Xag	-.05454257***	-.05454257***
N _Ico5Xag	.01258843	.01258843
S _Ico6Xag	-.04205787***	
_cons	52.185024***	52.185024***
N	21397	21397
r2	.01577418	.01577418
l1	-74852.777	-74852.777

legend: * p<0.05; ** p<0.01; *** p<0.001

A likelihood ratio test tells us, that the $CHI^2(5) = 139.02$. We need not look at a CHI^2 table to believe that this is highly significant. We should briefly recall how it is possible to compute the coefficient and its standard error for France (cat. 1) and for age of 18 using the coefficients from the first column of Table 49. Of course, this could be done easily by estimating the model a second time with a different omitted category for the countries, simultaneously centring the age at 18. However, the missing coefficient can be estimated by computing the negative sum of all the 5 conditional effects for country plus the negative sum of all the 5 interaction effects times 18:

$$\begin{aligned} & \text{display } -(_b[_Icountryn_2] + _b[_Icountryn_3] \\ & \quad + _b[_Icountryn_4] + _b[_Icountryn_5] \\ & \quad + _b[_Icountryn_6]) + \\ & (- (_b[_Ico2Xag] + _b[_Ico3Xag] + _b[_Ico4Xag] \\ & \quad + _b[_Ico5Xag] + _b[_Ico6Xag]) * 18) = \mathbf{.26632494} \end{aligned}$$

If we also want to get the standard error we define the selection vector :

$w = (1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 18 \ 18 \ 18 \ 18 \ 18 \ 0)$ and compute the standard error adopting Equation 2 and taking the square root of the resulting scalar. We will obtain **0.29584487** which indeed is what we wanted. Table 50 is the proof of the pudding!

Table 50: `xi3:regress mcs12 e.country*age18`

Source	SS	df	MS			
Model	21943.2666	11	1994.84242	Number of obs =	21397	
Residual	1369144.14	21385	64.0235747	F(11, 21385) =	31.16	
Total	1391087.41	21396	65.0162372	Prob > F =	0.0000	
				R-squared =	0.0158	
				Adj R-squared =	0.0153	
				Root MSE =	8.0015	

mcs12	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_Icountryn_1	.2663249	.2958449	0.90	0.368	-.3135532	.846203
_Icountryn_2	-1.842382	.2726636	-6.76	0.000	-2.376823	-1.307941
_Icountryn_3	-.5803438	.257397	-2.25	0.024	-1.084861	-.0758263
_Icountryn_4	1.120284	.220636	5.08	0.000	.6878214	1.552748
_Icountryn_5	.093804	.3062369	0.31	0.759	-.5064432	.6940512
age18	.0338543	.0034243	9.89	0.000	.0271423	.0405662
_Ico1Xag	.0216078	.0085591	2.52	0.012	.0048314	.0383842
_Ico2Xag	.0221729	.0081758	2.71	0.007	.0061478	.038198
_Ico3Xag	.0402313	.0075068	5.36	0.000	.0255175	.0549451
_Ico4Xag	-.0545426	.0065493	-8.33	0.000	-.0673798	-.0417054
_Ico5Xag	.0125884	.00888	1.42	0.156	-.004817	.0299939
_cons	52.7944	.1172794	450.16	0.000	52.56452	53.02428

Study suggestions 5: The reader is encouraged to try out different values of age. For instance, for 90 year old respondents you will be surprised !

3.3.3.3 Two categorical and one continuous predictor

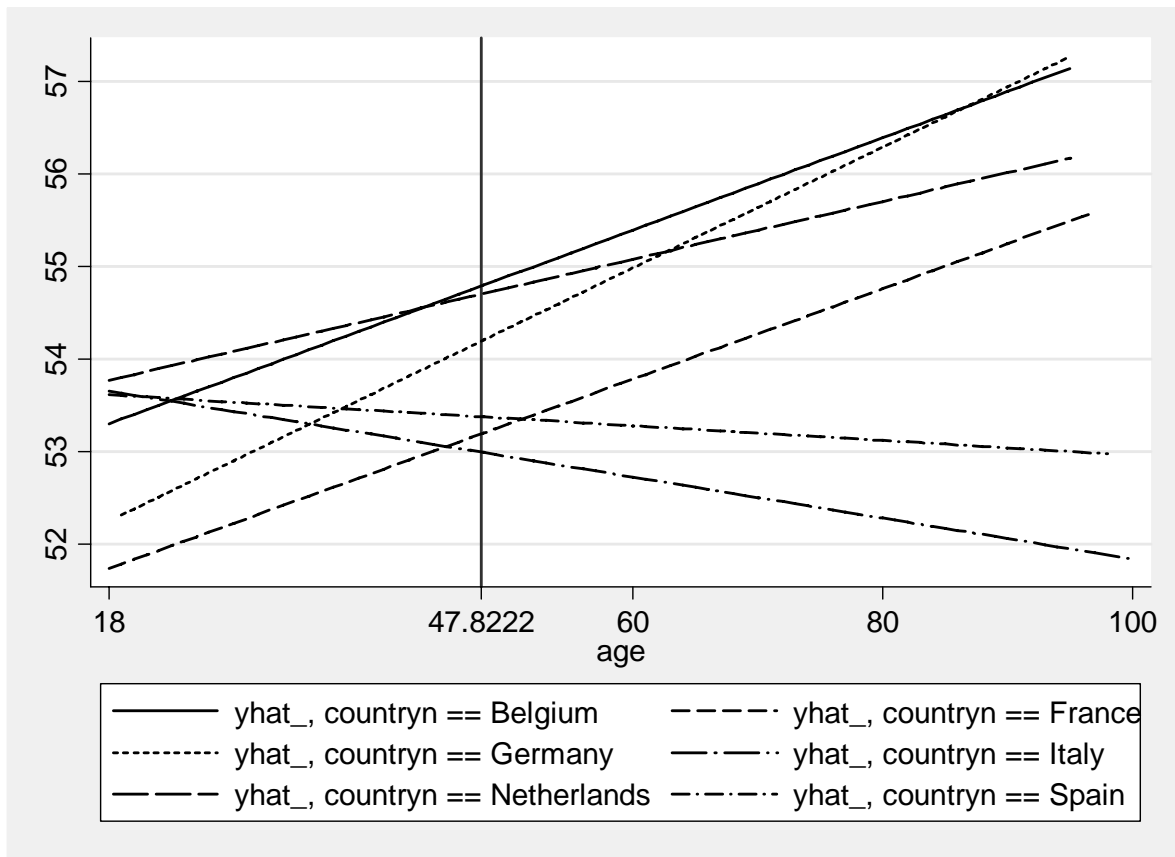
It might be of interest to further explore the relation between three variables, one of which is continuous. Adding a dichotomous variable “depression” (`dsm_mde`), the saturated model now also contains a three-way interaction. In the first step we will look at a model with Spain as “omitted category” and an additional predictor: `dsm_mde`. Plotting the predicted means will clearly show how the countries differ with respect to the effect of age on mental health related quality of life. In a second step we will estimate and interpret the three-way interaction.

Table 51: `xi3:regress mcs12 e.country*age_main dsm_mde`

e.country		_Icountry_1-6		(naturally coded; _Icountry_6 omitted)	
Source	SS	df	MS	Number of obs = 21397	
Model	140023.914	12	11668.6595	F(12, 21384) = 199.45	
Residual	1251063.5	21384	58.5046529	Prob > F = 0.0000	
Total	1391087.41	21396	65.0162372	R-squared = 0.1007	
				Adj R-squared = 0.1002	
				Root MSE = 7.6488	

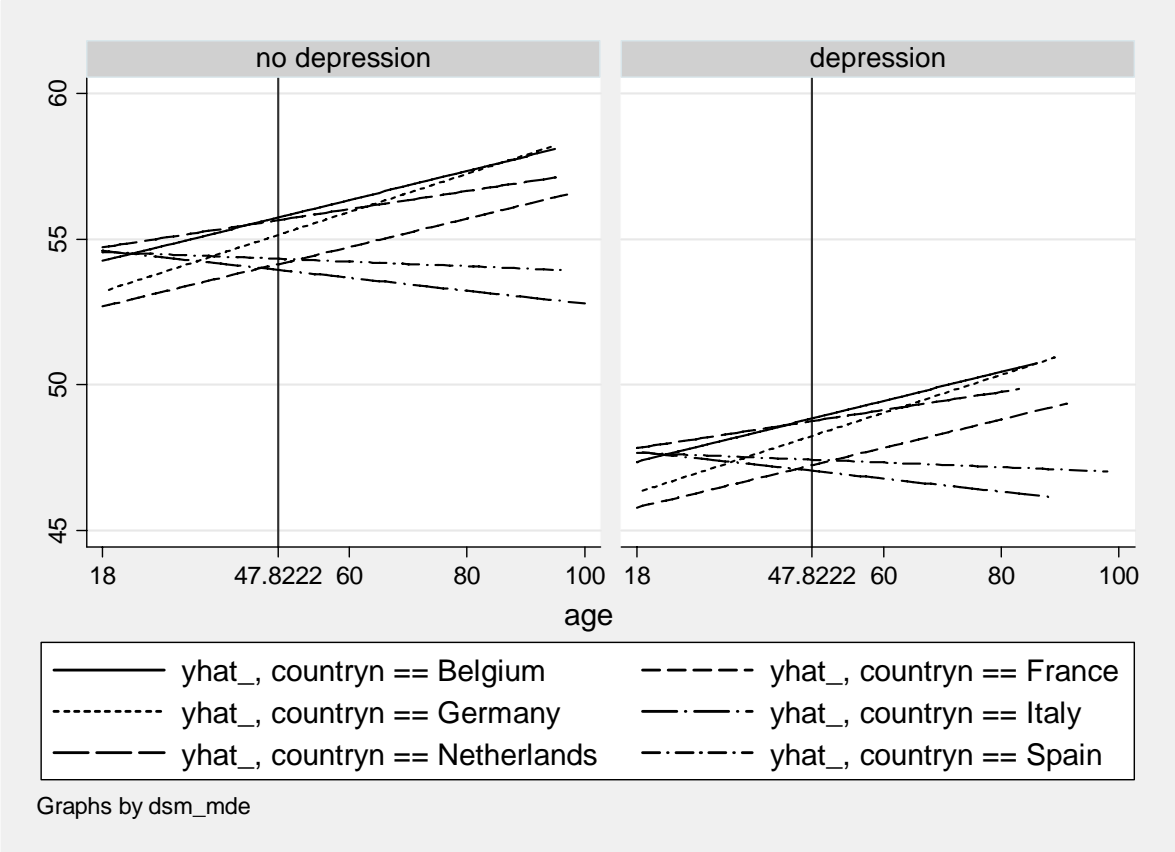
mcs12	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_Icountry_1	-.1560145	.4174662	-0.37	0.709	-.9742795	.6622506
_Icountry_2	-1.697964	.3894332	-4.36	0.000	-2.461282	-.9346455
_Icountry_3	-1.481953	.3646354	-4.06	0.000	-2.196666	-.7672407
_Icountry_4	1.490272	.3137112	4.75	0.000	.8753741	2.105169
_Icountry_5	.6505748	.4335755	1.50	0.134	-.1992657	1.500415
age_main	.0274432	.0032765	8.38	0.000	.021021	.0338654
_Ico1Xag	.0223619	.0081819	2.73	0.006	.0063248	.0383989
_Ico2Xag	.0212521	.0078155	2.72	0.007	.0059331	.036571
_Ico3Xag	.0376565	.0071761	5.25	0.000	.0235907	.0517223
_Ico4Xag	-.0495543	.0062617	-7.91	0.000	-.0618276	-.0372809
_Ico5Xag	.0036705	.0084909	0.43	0.666	-.0129724	.0203134
dsm_mde	-6.888887	.1533398	-44.93	0.000	-7.189444	-6.588329
_cons	53.51147	.1685913	317.40	0.000	53.18102	53.84192

Figure 2: `postgr3 age_main,by(countryn) scheme(s2mono) xlab(18 47.82217 60 80 100) xline(47.82217) two-way interaction`



If we look at the partial plot of the predicted values of `mcs12` against the age, disregarding the predictor `dsm_mde`, we see from Figure 2 that for Italy and Spain the effect of age on `mcs12` is considerably different. For these two countries the effect of age is negative, and it is left for the reader to compute both the conditional effect and the interaction term for Spain, which served as an omitted category. Of course, this prediction does not show the mean differences for depressive and non-depressive respondents, although this predictor is part of the model. The next Figure (Figure 3) shows the two-way interactions for both groups with and without depression, a graphical representation of the model from Table 51 or from Table 52. The particular centring of age is of no importance for both the model fit and the plot.

Figure 3: `postgr3 age_main, by(country) sep(dsm_mde) two-way interaction only`



The conditional effects for the countries (the first 5 parameters of Table 51) are meaningless, since they estimate and test these conditional differences for respondents not even born. One will ask quite legitimately why the effect for Italy is positive and for Germany negative, although Figure 2 suspects the difference to be just the other way around. As said before, the conditional effects for the countries test the deviation from the grand mean for the age of zero (!). We already computed the result for an age of 18 in Table 50, and from Figure 2 we can see that these conditional coefficients change dramatically if only the respondents approach

90. Table 52 shows the results for the regression if the age is centred at 90. Also compare results provided in Table 55.

Table 52: `xi3:regress mcs12 e.country*age90 dsm_mde`

mcs12	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_Icountry_1	1.856552	.3695631	5.02	0.000	1.132181	2.580924
_Icountry_2	.2147221	.3599286	0.60	0.551	-.4907648	.9202091
_Icountry_3	1.907129	.3235741	5.89	0.000	1.272899	2.541358
_Icountry_4	-2.969612	.2889784	-10.28	0.000	-3.536031	-2.403192
_Icountry_5	.9809207	.3803656	2.58	0.010	.2353757	1.726466
age90	.0274432	.0032765	8.38	0.000	.021021	.0338654
_Ico1Xag	.0223619	.0081819	2.73	0.006	.0063248	.0383989
_Ico2Xag	.0212521	.0078155	2.72	0.007	.0059331	.036571
_Ico3Xag	.0376565	.0071761	5.25	0.000	.0235907	.0517223
_Ico4Xag	-.0495543	.0062617	-7.91	0.000	-.0618276	-.0372809
_Ico5Xag	.0036705	.0084909	0.43	0.666	-.0129724	.0203134
dsm_mde	-6.888887	.1533398	-44.93	0.000	-7.189444	-6.588329
_cons	55.98136	.1495047	374.45	0.000	55.68832	56.2744

estimates store moint

Table 53: `xi3:regress mcs12 e.country*age_main*dsm_mde`

Source	SS	df	MS	Number of obs =	21397
Model	142550.414	23	6197.84408	F(23, 21373) =	106.10
Residual	1248537	21373	58.4165535	Prob > F =	0.0000
				R-squared =	0.1025
				Adj R-squared =	0.1015
Total	1391087.41	21396	65.0162372	Root MSE =	7.6431

mcs12	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_Icountry_1	-.0009122	.445809	-0.00	0.998	-.8747313	.8729068
_Icountry_2	-1.693102	.4344548	-3.90	0.000	-2.544666	-.8415378
_Icountry_3	-1.406522	.387742	-3.63	0.000	-2.166526	-.6465189
_Icountry_4	1.493493	.3306763	4.52	0.000	.8453425	2.141643
_Icountry_5	.3571543	.4759232	0.75	0.453	-.5756908	1.289999
age_main	.0279851	.0035087	7.98	0.000	.0211079	.0348624
dsm_mde	-6.697177	.4995717	-13.41	0.000	-7.676375	-5.71798
_Ico1Xag	.0169281	.0086685	1.95	0.051	-.0000628	.0339189
_Ico2Xag	.0193265	.0086491	2.23	0.025	.0023737	.0362794
_Ico3Xag	.0360204	.0075703	4.76	0.000	.0211819	.0508588
_Ico4Xag	-.046617	.0065775	-7.09	0.000	-.0595094	-.0337247
_Ico5Xag	.0072775	.0091802	0.79	0.428	-.0107164	.0252714
_Ico1Xds	-1.723186	1.2987	-1.33	0.185	-4.268736	.8223636
_Ico2Xds	-.0819495	1.002943	-0.08	0.935	-2.047792	1.883893
_Ico3Xds	-.5778693	1.173066	-0.49	0.622	-2.877166	1.721427
_Ico4Xds	.8766869	1.078035	0.81	0.416	-1.236343	2.989717
_Ico5Xds	1.376834	1.19349	1.15	0.249	-.9624951	3.716164
_IagXds	-.0012807	.0102531	-0.12	0.901	-.0213776	.0188163
_Ico1XagXds	.0557987	.0265154	2.10	0.035	.0038265	.1077709
_Ico2XagXds	.0116232	.0206752	0.56	0.574	-.0289018	.0521482
_Ico3XagXds	.0197956	.0245222	0.81	0.420	-.0282698	.067861
_Ico4XagXds	-.0442625	.0220638	-2.01	0.045	-.0875092	-.0010158
_Ico5XagXds	-.0153678	.0247549	-0.62	0.535	-.0638894	.0331537
_cons	53.4427	.1794886	297.75	0.000	53.09088	53.79451

estimates store mo3int

Table 54: `lrtest moint mo3int`

Likelihood-ratio test	LR chi2(12) =	1973.09
(Assumption: moint nested in mo3int)	Prob > chi2 =	0.0000

Now the conditional effect for Italy (`_Icountry_4`) is significantly **below** the average, which can be immediately detected from Figure 2. Whatever centring we choose, further interaction terms tell us that the results shown until now are different for the two subgroups with and without depression. Although the increase in explained variance is only marginal, the LR test is in favour of the model with all possible coefficients. Since a linear effect is adopted for age assuming the proportionality of this effect for each year, this model is not a saturated model ! In Table 53 the coefficients are provided, but without plotting the predicted values for the two groups, as in Figure 3, results are hardly interpretable. Particularly the conditional effects `_Icountry_1` to `_Icountry_5` describe the differences between the countries for those without depression (coded zero) and the age of zero, which is of no interest at all.

Figure 4: `postgr3 age_main,by(countryn) scheme(s2mono) xlab(18 47.82217 60 80 100) xline(47.82217) sep(dsm_mde) three-way interaction`

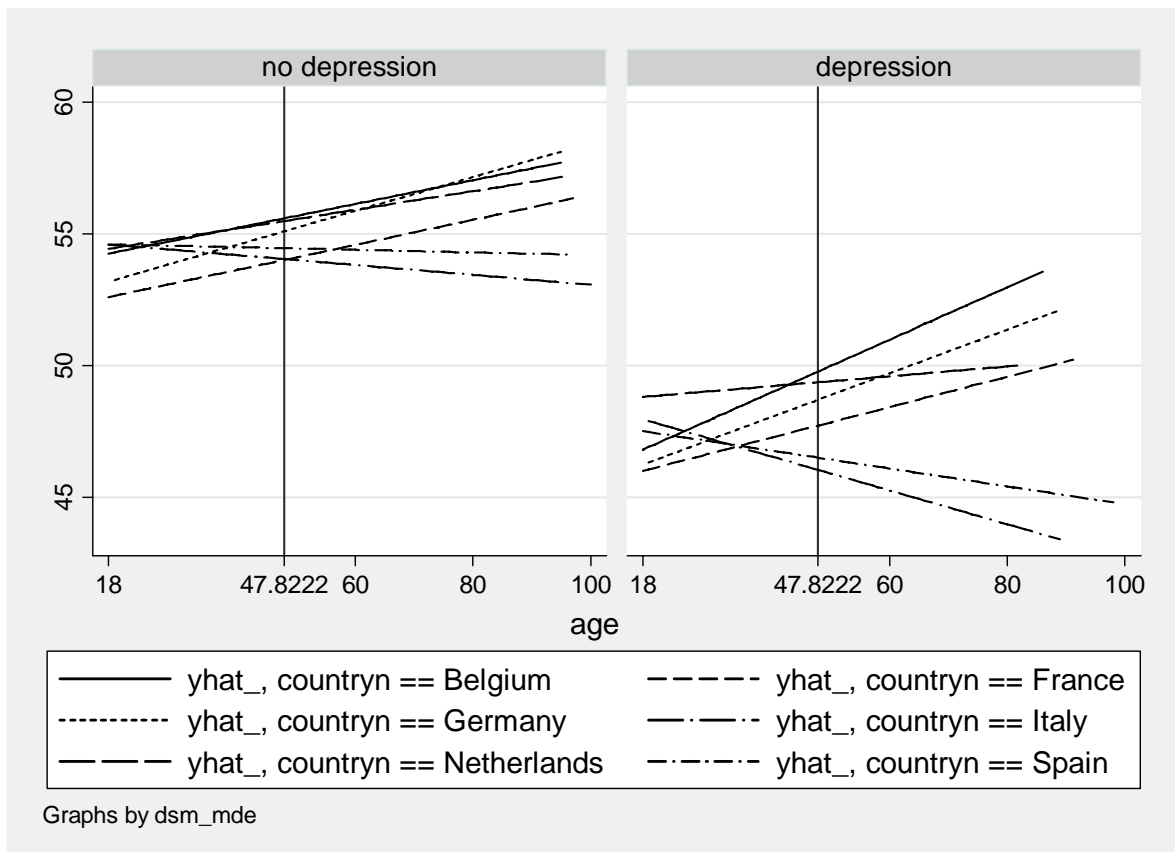


Figure 4 shows that the differences between the 6 countries are much more prominent for those respondents who suffer from life-time depression. For some reasons, Spain and Italy differ even more from all the other nations if the respondents become older and suffer from depression. The biggest difference is observed between respondents from Belgium and Italy for those over 80 years of age. For people younger than 40 the difference between the country seems to be negligible. Only the French (at least those covered by the sample at hand) seem to be an exception.

Results presented above seem to indicate what is called a cohort effect rather than an effect of age “as is”. The differences between the different cultures in Europe are smaller for younger people than for those born at the end of the 2nd war or even earlier. Why these differences are even more prominent for those suffering from depression could not be answered here. Even though the LR test is in favour of the 3-way interaction model, we clearly see that there are no essential differences between the two groups indicated by `dsm_mde`. In a last step we should have a look at all the parameters tabulated next to each other, where only the age is centred for different values. Table 55 presents all the necessary information to see which parameters depend on the centring and how this change must be interpreted.

We see immediately that only those coefficients change systematically, which do not contain the age as one term in the product. All the other coefficients obviously depend on the centring of age, because they are conditional effects depending on one particular value (zero) of age. We could ask for the meaning of two further contrasts independent from the centring of age: `age*dsm_mde` called `_IagXds` in the output. This is the interaction between age and depression for an “average” country, since we employed **effect** coding for the countries. The effect is rather small and we would not see very much from plotting this effect by means of the `postgr3` command. This is an exercise left for the reader.

Table 55: estimates table mo3int age18 age40 agemean age50 age90 ,star Country: unweighted effect coding; dsm_mde: dummy coded

Variable	age0	age18	age40	agemean	age50	age90
_Icountry_1	-.00091222	.30379273	.6762099***	.80862403***	.84549042***	1.5226125***
_Icountry_2	-1.6931019***	-1.345224***	-.92003996***	-.76886439***	-.72677448***	.04628744
_Icountry_3	-1.4065224***	-.75815592**	.03429197	.3160494*	.39449556**	1.8353099***
_Icountry_4	1.4934929***	.65438675**	-.37118741**	-.73583363***	-.83735748***	-2.7020378***
_Icountry_5	.3571543	.48814898	.64825358***	.7051793***	.7210284***	1.0121277*
dsm_mde	-6.6971775***	-6.7202292***	-6.7484035***	-6.758421***	-6.76121***	-6.8124361***
_Ico1Xag	.01692805	.01692805	.01692805	.01692805	.01692805	.01692805
_Ico2Xag	.01932655*	.01932655*	.01932655*	.01932655*	.01932655*	.01932655*
_Ico3Xag	.03602036***	.03602036***	.03602036***	.03602036***	.03602036***	.03602036***
_Ico4Xag	-.04661701***	-.04661701***	-.04661701***	-.04661701***	-.04661701***	-.04661701***
_Ico5Xag	.00727748	.00727748	.00727748	.00727748	.00727748	.00727748
_Ico1Xds	-1.7231864	-.7188092	.50876291	.94523016*	1.0667502**	3.2986995**
_Ico2Xds	-.08194949	.12726845	.38297926	.47389808	.49921145	.96414021
_Ico3Xds	-.5778693	-.22154816	.21395544	.36880016	.41191163	1.2037364
_Ico4Xds	.87668694	.07996216	-.89381257*	-1.2400413***	-1.3364374***	-3.106937**
_Ico5Xds	1.3768345	1.1002134	.76212105*	.64191121	.60844269	-.00627074
_IagXds	-.00128065	-.00128065	-.00128065	-.00128065	-.00128065	-.00128065
_Ico1XagXds	.05579873*	.05579873*	.05579873*	.05579873*	.05579873*	.05579873*
_Ico2XagXds	.01162322	.01162322	.01162322	.01162322	.01162322	.01162322
_Ico3XagXds	.01979562	.01979562	.01979562	.01979562	.01979562	.01979562
_Ico4XagXds	-.04426249*	-.04426249*	-.04426249*	-.04426249*	-.04426249*	-.04426249*
_Ico5XagXds	-.01536784	-.01536784	-.01536784	-.01536784	-.01536784	-.01536784
age0	.02798513***					
age18		.02798513***				
age40			.02798513***			
agemean				.02798513***		
age50					.02798513***	
age90						.02798513***
_cons	53.442695***	53.946428***	54.5621***	54.781005***	54.841952***	55.961357***

legend: * p<0.05; ** p<0.01; *** p<0.001

4 Adjacent and Helmert coding

These two forms of contrasts are useful if the predictor can be seen as an ordinal one. Both contrasts are rather similar. The main difference with respect to dummy or effect coding is that the reference category changes for each contrast. This also means that the choice of a reference category is of minor importance, since it does not change the structure of the contrast. Both contrasts exist, as it is called, “forward” and “backward”, but we will focus on “forward” contrasts only, as “backward” coding might by itself explain and the reader is encouraged to estimate this as an exercise. In chapter 6 we will show how these coding schemes fit into a more general scheme of one-way designs in linear models.

4.1 Forward Adjacent coding

The purpose of this coding system is to compare the mean of a criterion for each level (category) of a predictor with the mean regarding the subsequent level (category) of this predictor. For an example we now use the 4 categories of *skt* as outlined in section 1. The allocation of numbers depends on the number of categories, so for a 4 category variable we need to build 3 indicator variables, according to the following scheme:

Table 56: Design matrix for forward adjacent coding

	Categories of <i>skt</i>			
	1	2	3	4
A_1 (1 v 2)	.75	-.25	-.25	-.25
A_2 (2 v 3)	.5	.5	-.5	-.5
A_3 (3 v 4)	.25	.25	.25	-.75

Each column of this scheme indicates the weighting coefficients for the linear combination of each cell. The weighting coefficients now are not only zeros or ones (or $-1/1$) as for dummy and effect coding, but rather a linear combination of proportions which sum up to zero, both for each row and for the row containing the sum total of each column. In section 6.2 (page 67) we will explain the construction of the design matrix in more detail, providing the relevant references. The new variables can be generated by using the prefix `a` (`a.skt`), again employing the command `xi3`. Tabulating these indicator variables against the predictor of interest will further explain the design.

Table 57: Cross tabulation between the indicator variables for Adjacent coding and `skt`

`tab _Iskt_1 skt`

skt(1 vs. 2)	skt				Total
	1.00	2.00	3.00	4.00	
-.25	0	1,816	995	703	3,514
.75	1,511	0	0	0	1,511
Total	1,511	1,816	995	703	5,025

`tab _Iskt_2 skt`

skt(2 vs. 3)	skt				Total
	1.00	2.00	3.00	4.00	
-.5	0	0	995	703	1,698
.5	1,511	1,816	0	0	3,327
Total	1,511	1,816	995	703	5,025

`tab _Iskt_3 skt`

skt(3 vs. 4)	skt				Total
	1.00	2.00	3.00	4.00	
-.75	0	0	0	703	703
.25	1,511	1,816	995	0	4,322
Total	1,511	1,816	995	703	5,025

The coding scheme follows a simple algorithm, which is outlined in Table 58. The general form of this scheme is presented again for only 4 categories, but can be generalized easily to more than 3 contrasts (rows). The user should keep in mind that the names of the variables generated by `xi3` and one of the prefixes are not different for each contrast; only the labels are which do never show up in the output !

Table 58: Design matrix for forward adjacent coding (k categories)

	1	2	3	4
A_1 (1 v 2)	$(k-1)/k$	$-1/k$	$-1/k$	$-1/k$
A_2 (2 v 3)	$(k-2)/k$	$(k-2)/k$	$-2/k$	$-2/k$
A_3 (3 v 4)	$(k-3)/k$	$(k-3)/k$	$(k-3)/k$	$-3/k$

4.1.1 One 4-categorical predictor

First, let's have a look at the means of `dist1sum` for each of the 4 categories of `skt`.

Table 59: `tab skt, sum(dist1sum)`

```

Summary of egen
dist1sum=rsum(q15a-q15g) if
distmiss==0
skt |      Mean   Std. Dev.   Freq.
-----+-----
 1.00 |  15.788732  6.2545796   1491
 2.00 |  18.469376  6.2721614   1796
 3.00 |  15.380424  6.7361216    991
 4.00 |  16.633238  6.5488024    698
-----+-----
Total |  16.793408  6.5319258   4976

```

Table 60: `xi3:regress dist1sum a.skt`

```

Source |      SS      df      MS              Number of obs =   4976
-----+-----+-----+-----              F( 3, 4972) =   69.53
Model |  8546.16828    3  2848.72276              Prob > F      =   0.0000
Residual | 203717.456  4972  40.9729396              R-squared     =   0.0403
-----+-----+-----+-----              Adj R-squared =   0.0397
Total | 212263.624  4975  42.666055              Root MSE     =   6.401

-----+-----+-----+-----+-----+-----
dist1sum |      Coef.   Std. Err.    t    P>|t|    [95% Conf. Interval]
-----+-----+-----+-----+-----+-----
A_1  _Iskt_1 | -2.680644   .2242624   -11.95  0.000   -3.120297   -2.240991
A_2  _Iskt_2 |  3.088953   .2532953   12.20  0.000    2.592382    3.585523
A_3  _Iskt_3 | -1.252814   .3162997    -3.96  0.000   -1.872901   -.6327269
      _cons | 16.56794    .096934   170.92  0.000   16.37791   16.75798
-----+-----+-----+-----+-----+-----

```

It is easy to see that the first coefficient represents the difference between category 1 and 2; the second coefficient the difference between 2 and 3, etc. Again, the *grand mean* is represented by the constant.

`display (15.788732+18.469376+15.380424+16.633238)/4` = **16.567943**

We should now try to compute the (predicted) mean for each of the 4 cells just as we have done before. We can use the syntax of STATA and type:

`display _b[_cons]+_b[_Iskt_1]*0.75+_b[_Iskt_2]*0.5+_b[_Iskt_3]*0.25`

15.788732

`display _b[_cons]+_b[_Iskt_1]*-0.25+_b[_Iskt_2]*0.5+_b[_Iskt_3]*0.25`

18.469376

`display _b[_cons]+_b[_Iskt_1]*-0.25+_b[_Iskt_2]*-0.5+_b[_Iskt_3]*0.25`

15.380424

`display _b[_cons]+_b[_Iskt_1]*-0.25+_b[_Iskt_2]*-0.5+_b[_Iskt_3]*-0.75`

16.633238

As expected, we obtain the exact means for the 4 cells (comp. Table 59), since the model has only one predictor and is saturated by default. If there is no access to this feature of STATA, we have to plug in the values for `b` instead of typing `_b[_cons]+_b[_Iskt_1]`, `_b[_Iskt_2]` and `_b[_Iskt_3]`.

4.1.2 Two categorical predictors

For this example we again employ the unrestricted variable `skt` just as described in section 1. The actual eight cell means of `dist1sum` can be found in Table 61. The models will contain a mixture of adjacent coded `skt` and dummy coded `q0rec`.

Table 61: `tab skt q0rec, sum(dist1sum)`

skt	q0rec		Total
	0	1	
1.00	18.349177	14.305085	15.788732
	5.7618022	6.0484204	6.2545796
	547	944	1491
2.00	19.593592	16.283607	18.469376
	5.9733664	6.2671683	6.2721614
	1186	610	1796
3.00	16.798762	14.694611	15.380424
	6.9977635	6.5010322	6.7361216
	323	668	991
4.00	17.371571	15.636364	16.633238
	6.537514	6.4417156	6.5488024
	401	297	698
Total	18.586488	15.044462	16.793408
	6.256263	6.3185563	6.5319258
	2457	2519	4976

Two hierarchically nested models using the last category as omitted category were estimated. Predicted means for the first model without interactions is presented in Table 63.

Table 62: `xi3:regress distlsum a.skt q0rec`

```
a.skt          _Iskt_1-4          (naturally coded; _Iskt_4 omitted)
```

Source	SS	df	MS	Number of obs = 4976		
Model	19242.248	4	4810.562	F(4, 4971)	=	123.89
Residual	193021.376	4971	38.8294862	Prob > F	=	0.0000
				R-squared	=	0.0907
				Adj R-squared	=	0.0899
				Root MSE	=	6.2313

distlsum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_Iskt_1	-1.780671	.2249509	-7.92	0.000	-2.221674	-1.339668
_Iskt_2	2.063455	.2542043	8.12	0.000	1.565103	2.561808
_Iskt_3	-.4905968	.3113211	-1.58	0.115	-1.100924	.11973
q0rec	-3.066468	.1847596	-16.60	0.000	-3.428679	-2.704258
_cons	18.15664	.1344144	135.08	0.000	17.89313	18.42015

```
predict distsuma if e(sample)
(option xb assumed; fitted values)
(49 missing values generated)
```

Table 63: `tab skt q0rec, sum(distsuma)`

skt	q0rec		Total
	0	1	
1.00	17.730211	14.663744	15.788732
	.	.	1.4783799
	547	944	1491
2.00	19.510883	16.444414	18.469377
	.	.	1.4526483
	1186	610	1796
3.00	17.447428	14.38096	15.380424
	.	.	1.4380485
	323	668	991
4.00	17.938025	14.871556	16.633238
	.	.	1.5172068
	401	297	698
Total	18.586488	15.044462	16.793408
	.90290254	.80588189	1.9666694
	2457	2519	4976

Adopting the predicted values (distsuma) conditioned on q0rec==0, we obtain the constant:

```
display (17.730211+19.510883+17.447428+17.938025)/4 = 18.15664
```

Estimating the difference between the two grand means, we obtain:

```
display ((14.663744+16.444414+14.38096+14.871556)/4)-
        ((17.730211+19.510883+17.447428+17.938025)/4) = -3.0664683
```

As in previous chapters the model with and without interaction effects can be compared by storing and tabulating using the `estimates` command. The two models were called `mo1` and `mo2`, without and with interactions, respectively. By employing the `table` feature of `estimates`, we obtain in Table 64:

Table 64: `estimates table mo1 mo2,star stats(N ll r2 bic aic)`

Variable	mo1	mo2	
b1 _Iskt_1	-1.7806709***	-1.2444146***	A_1
b2 _Iskt_2	2.0634552***	2.7948303***	A_2 q0rec = 0
b3 _Iskt_3	-.49059681	-.57280946	A_3
b4 q0rec	-3.0664683***	-2.7983591***	
b5 _Isk1Xq0		-.73410724	
b6 _Isk2Xq0		-1.2058345*	
b7 _Isk3Xq0		-.3689434	
b0 _cons	18.156637***	18.028275***	
N	4976	4976	
ll	-16162.177	-16151.081	
r2	.09065259	.09469899	
bic	32366.915	32370.261	
aic	32334.353	32318.161	

legend: * p<0.05; ** p<0.01; *** p<0.001

Table 65: `lrtest mo1 mo2`

```
Likelihood-ratio test
(Assumption: mo1 nested in mo2)
LR chi2(3) = 22.19
Prob > chi2 = 0.0001
```

Although the likelihood ratio test declares the two models to be statistically different, only one effect turns out to be significant (`_Isk2Xq0`); the change in R^2 is negligible and the BIC also allow you to prefer the more parsimonious model presented in Table 62. However, we should have a closer look at the meaning of both conditional effects and interaction terms in the case of forward adjacent coding of the row defining predictor `skt`. Using the predicted means from the fully saturated model (which are, of course, the actual means of Table 61), we

see that the difference between category 2 and 3 is 2.7984 if $q_0 = 0$. This difference is 1.588996 if $q_0 = 1$ which can be computed from Table 61.

$$\Delta \text{skt}_2, \text{skt}_3 | q_0 \text{rec} = 1 = b_2 + b_6$$

If no interaction effect is employed these difference would be equal (b_2 of m_01 : 2.0634552); the difference generated by the vignette is **-1.205834** which is represented and tested by the interaction parameter just mentioned before (b_6 of m_02).

Study suggestions 6: Verify the meaning of $q_0 \text{rec}$ in model m_02 of Table 64.

4.2 Forward Helmert coding

The idea of the so called Helmert contrast is to compare each level of categorical variable with the *mean* of all subsequent levels. This is similar to adjacent coding but makes sense only if we really expect the variable to be ordinal and do not want to inspect whether this assumption really holds. It should be employed with care if we must assume that the different levels of the predictor are ordinal in their own nature but not necessarily with respect to the criterion used as dependent variable. For sake of simplicity we will again use our variable skt (with 4 levels). The coding scheme is only of minor difference and looks like:

Table 66: Design matrix for forward Helmert coding (4 categories)

	1	2	3	4
H_1 (1 v 2+)	.75	-.25	-.25	-.25
H_2 (2 v 3+)	.0	0.66	-.33	-.33
H_3 (3 v 4)	0	0	0.5	-0.5

We see that the sum of each row is zero and, furthermore, the sum of the sums of all columns is also zero – again all effects are orthogonal on each other. Tabulating the indicators H_1 to H_3 against the original variable may facilitate the “meaning” of these predictors. It must be

mentioned, that the variable names created by `xi3` do not differ, and only the labels tell the user what kind of a contrast was employed.

Table 67: Cross tabulation between the indicator variables for Helmert coding and `skt`

`tab _Iskt_1 skt`

skt(1 vs. 2+)	skt				Total
	1.00	2.00	3.00	4.00	
-.25	0	1,816	995	703	3,514
.75	1,511	0	0	0	1,511
Total	1,511	1,816	995	703	5,025

`tab _Iskt_2 skt`

skt(2 vs. 3+)	skt				Total
	1.00	2.00	3.00	4.00	
-.3333333	0	0	995	703	1,698
0	1,511	0	0	0	1,511
.6666667	0	1,816	0	0	1,816
Total	1,511	1,816	995	703	5,025

`tab _Iskt_3 skt`

skt(3 vs. 4)	skt				Total
	1.00	2.00	3.00	4.00	
-.5	0	0	0	703	703
0	1,511	1,816	0	0	3,327
.5	0	0	995	0	995
Total	1,511	1,816	995	703	5,025

The more general coding scheme for k levels of the predictor is shown in Table 68. This example demonstrate the coding scheme again for an example of 4 categories, but can easily be expanded to k categories :

Table 68: Design matrix for forward Helmert coding (k categories)

	1	2	3	4
H_1 (1 v 2+)	$(k-1)/k$	$-1/k$	$-1/k$	$-1/k$
H_2 (2 v 3+)	.0	$((k-1)-1)/(k-1)$	$-1/(k-1)$	$-1/(k-1)$
H_3 (3 v 4)	0	0	$((k-2)-1)/(k-2)$	$-1/(k-2)$

4.2.1 One 4- categorical predictor

We employ the same data as in section 4.1.1. The means for each category of `skt` can be found in Table 59. The necessary indicator variables can be obtained by typing `h.[varname]` and using `xi3` just as before.

Table 69: `xi3:regress dist1sum h.skt`

h.skt		_Iskt_1-4			(naturally coded; _Iskt_4 omitted)		
Source	SS	df	MS	Number of obs = 4976			
Model	8546.16828	3	2848.72276	F(3, 4972) = 69.53			
Residual	203717.456	4972	40.9729396	Prob > F = 0.0000			
Total	212263.624	4975	42.666055	R-squared = 0.0403			
				Adj R-squared = 0.0397			
				Root MSE = 6.401			

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_Iskt_1	-1.038947	.2028082	-5.12	0.000	-1.436541	-.6413534
_Iskt_2	2.462546	.2186889	11.26	0.000	2.033819	2.891272
_Iskt_3	-1.252814	.3162997	-3.96	0.000	-1.872901	-.6327269
_cons	16.56794	.096934	170.92	0.000	16.37791	16.75798

If we calculate the grand mean of the 4 levels, we obtain:

$$(15.788732+18.469376+15.380424+16.633238)/4 = \mathbf{16.567943}$$

and furthermore:

$$(18.469376 + 15.380424 + 16.633238)/3 - 15.788732 = \mathbf{-1.038947}$$

which is just equal to the first parameter (`_Iskt_1`) of the model below.

The reader will certainly believe that the other parameters represent the expected differences. The example presented above is not very exciting, and we should turn immediately to an example with two predictors again; with and without interaction.

4.2.2 Two categorical predictors

We will see that the 2 different coding systems (adjacent and Helmert) provide the same coefficients for the constant and the conditional effect of the dichotomous, dummy coded predictor. Without interactions, the conditional effect of `q0rec` holds for each column; this should not be stressed in the following, since it adds nothing new to what has been said already. On the other hand, it sheds light on the advantage of all the last contrasts over the

ordinary dummy coding: Whatever reference (omitted) category is chosen, **conditional** effects keep the same indicating differences of grand means. If one wants to look back again, the cell means of interest are provided in Table 61. The predicted values from this model are the same as for adjacent coding and can be found in Table 63. We estimated both a model with and without interaction, stored them under the names h1 and h2 respectively, and tabulate the results next to each other (Table 70). Commands are written as follows:

```
xi3:regress dist1sum h.skt q0rec
predict dist1sumh if e(sample)
estimates store h1
xi3:regress dist1sum h.skt*q0rec
estimates store h2
```

Table 70. `estimates table h1 h2,stats(r2 N)star`

Variable	h1	h2
_Iskt_1	-.56856636**	.42786913
_Iskt_2	1.8181568***	2.5084256***
_Iskt_3	-.49059681	-.57280946
q0rec	-3.0664683***	-2.7983591***
_Isk1Xq0		-1.660978***
_Isk2Xq0		-1.3903062**
_Isk3Xq0		-.3689434
_cons	18.156637***	18.028275***
r2	.09065259	.09469899
N	4976	4976

legend: * p<0.05; ** p<0.01; *** p<0.001

lrtest h1 h2

```
Likelihood-ratio test                                LR chi2(3) =      22.19
(Assumption: h1 nested in h2)                       Prob > chi2 =      0.0001
```

Again, the effect for q0rec and the constant must differ, since only the model h2 is a saturated one. The `lrtest` tells that the interaction is statistically necessary. We will find the predicted values from the model h1 (without interaction effect) in Table 71. Computing the first contrast condition for both vignettes, we find, as expected, the same value.

```
display 17.730211 -((19.510883+17.447428+17.938025)/3)      -.56856767
display 14.663744 -((16.444414+14.38096 +14.871556)/3)     -.568566
```

Table 71: `tab skt q0rec, sum(dist1sumh)`

Means, Standard Deviations and Frequencies of Fitted values

skt	q0rec		Total
	0	1	
1.00	17.730211	14.663744	15.788732
	.	.	1.4783799
	547	944	1491
2.00	19.510883	16.444414	18.469377
	.	.	1.4526483
	1186	610	1796
3.00	17.447428	14.38096	15.380424
	.	.	1.4380485
	323	668	991
4.00	17.938025	14.871556	16.633238
	.	.	1.5172068
	401	297	698
Total	18.586488	15.044462	16.793408
	.90290254	.80588189	1.9666694
	2457	2519	4976

For the saturated model things become entirely different. Taking cell means again from Table 61, we now observe two very different contrasts for the values of q0rec (Schizophrenia versus depression):

Schizophrenia `display` $18.349177 - ((19.593592 + 16.798762 + 17.371571) / 3)$ **.42786867**

Depression `display` $14.305085 - ((16.283607 + 14.694611 + 15.636364) / 3)$ **-1.233109**

Of course, the difference between these two contrasts is -1.660978 , which is nothing but the significant interaction parameter `_Isk1Xq0` from Table 70. Again, the results show that in this case respondents are presented with a schizophrenic episode, those who label the person described as “schizophrenic” show a more pronounced social distance than all the other. For those who had to label the vignette “Depression”, things are just the other way round. Therefore, looking only on labelling would only tell us half of the story – it is of great importance **what symptoms** were offered to be evaluated by the respondents.

5 Summary and conclusion

Here we once again have a look at all contrasts next to each other. The second part of Table 72 only confirms that the different contrasts used are all statistically equivalent. This holds, although the model parameters may differ considerably. Modelnames are chosen arbitrarily due to the meaning of the contrast.

Table 72: estimates table I E A H,star stats(N ll r2 bic aic) b(%10.3f) The capital letter indicates the type of contrast.

VARIABLE	I	E	A	H
_Iskt_1	0.978*	0.321	-1.244***	0.428
_Iskt_2	2.222***	1.565***	2.795***	2.508***
_Iskt_3	-0.573	-1.230***	-0.573	-0.573
q0rec	-1.735***	-2.798***	-2.798***	-2.798***
_Isk1Xq0	-2.309***	-1.246***	-0.734	-1.661***
_Isk2Xq0	-1.575**	-0.512	-1.206*	-1.390**
_Isk3Xq0	-0.369	0.694	-0.369	-0.369
_cons	17.372***	18.028***	18.028***	18.028***
N	4976.000	4976.000	4976.000	4976.000
ll	-16151.081	-16151.081	-16151.081	-16151.081
r2	0.095	0.095	0.095	0.095
bic	32370.261	32370.261	32370.261	32370.261
aic	32318.161	32318.161	32318.161	32318.161

Legend: * p<0.05; ** p<0.01; *** p<0.001

6 User defined coding

In this last chapter two related goals will be pursued. First we want to show how the contrasts, e.g. the contrast- or indicator variables are actually built, and secondly how it is possible to define ones own contrasts, tailored to a particular research question. We decided to deal with this problems at the end of this textbook, simply because the average user never has to carry out all the following procedures manually, but rather employ one of the predefined contrasts using the program adopted for all the analyses above. In any case, it is of great importance to see how the contrast schemes are transformed to design matrices, which afterwards are employed to generate the predictors. We will briefly outline this procedure for two examples already presented above, and one arbitrarily chosen contrast.

To introduce the problem we will briefly summarize the linear one-way model (compare (Finn, 1974: 219pp and 235pp) for a 4- categorical predictor. The model may be written as :

$$y. = A\theta + E$$

We notate $y.$ instead of y because usually more than one person is observed for each level.

$$y. = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{pmatrix}$$

The matrix A is only of rank 4, since the last column is exactly the first column minus the sum of the remaining ones (2,3, and 4). Therefore, only 4 contrasts for θ are uniquely estimable. It is necessary to define linear combinations of parameters by the rows of a so called contrast matrix L. The model matrix A can be defined as the product of the contrast matrix L and the design matrix frequently called K

$$A = KL$$

In order to obtain the matrix of interest K, we have to compute:

$$AL' = KLL'$$

$$AL' (LL')^{-1} = K (LL') (LL')^{-1}$$

$$K = AL' (LL')^{-1}$$

It was shown by Bock (Bock, 1963) that K can be constructed without reference to A. The necessary basis may be calculated from

$$K_a = [1, K_{ca}] \quad \text{and therefore}$$

$$K_{ca} = L_{ca}' (L_{ca} L_{ca}')^{-1}$$

For sake of simplicity we will let out all subscripts and use L and K respectively. What we have to do is define L, where each row defines one contrast.

6.1 "Dummy coding"

Of course, even ordinary dummy coding can be defined by means of this scheme outlined above. To do this we first have to look at the coding scheme more closely, although it looks so trivial at first glance. The contrast matrix has the form:

Table 73: Contrast matrix L for dummy coding

	c1	c2	c3	c4
r1	1	-1	0	0
r2	1	0	-1	0
r3	1	0	0	-1

Writing:

```
matrix input L = (1 -1 0 0 \  
                  1 0 -1 0 \  
                  1 0 0 -1)
```

and matrix $K = L' * inv(L * L')$ we get the necessary information to compute the 3 design variables. Each row portrays the contrast between one of the 3 categories with the reference category (here the category 1 was decided to serve as omitted category). It is not a 0 and 1 matrix anymore, but rather employs a weighting with respect to the number of categories, similar to what is done in the frame of Adjacent – and Helmert coding.

Table 74: Design matrix K for dummy coding

	d1	d2	d3
c1	.25	.25	.25
c2	-.75	.25	.25
c3	.25	-.75	.25
c4	.25	.25	-.75

This matrix provides the necessary information to calculate the three necessary predictors. These predictors (now we call it d1, d2 and d3) should estimate and test the differences, the same way as the i – option of the xi3 precommand does. The 3 variables are constructed from the coefficients of each column:

```
gen d1=.25 if skt!=2  
gen d2=.25 if skt!=3  
gen d3=.25 if skt!=4  
replace d1=-.75 if skt==2  
replace d2=-.75 if skt==3  
replace d3=-.75 if skt==4
```

Table 75: regress distlsum d1 d2 d3

Source	SS	df	MS			
Model	8546.16828	3	2848.72276	Number of obs =	4976	
Residual	203717.456	4972	40.9729396	F(3, 4972) =	69.53	
				Prob > F =	0.0000	
				R-squared =	0.0403	
				Adj R-squared =	0.0397	
Total	212263.624	4975	42.666055	Root MSE =	6.401	

distlsum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
d1	-2.680644	.2242624	-11.95	0.000	-3.120297	-2.240991
d2	.4083086	.2623456	1.56	0.120	-.1060046	.9226217
d3	-.8445054	.2935654	-2.88	0.004	-1.420023	-.2689877
_cons	16.56794	.096934	170.92	0.000	16.37791	16.75798

Table 76: xi3:regress distlsum i.skt

i.skt _Iskt_1-4 (naturally coded; _Iskt_1 omitted)

Source	SS	df	MS			
Model	8546.16828	3	2848.72276	Number of obs =	4976	
Residual	203717.456	4972	40.9729396	F(3, 4972) =	69.53	
				Prob > F =	0.0000	
				R-squared =	0.0403	
				Adj R-squared =	0.0397	
Total	212263.624	4975	42.666055	Root MSE =	6.401	

distlsum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_Iskt_2	2.680644	.2242624	11.95	0.000	2.240991	3.120297
_Iskt_3	-.4083086	.2623456	-1.56	0.120	-.9226217	.1060046
_Iskt_4	.8445054	.2935654	2.88	0.004	.2689877	1.420023
_cons	15.78873	.1657715	95.24	0.000	15.46375	16.11372

Comparing these results, we surprisingly see, that they are *not* identical, particularly not with respect to the constant ! The constant now is not the mean for the omitted category, but rather the grand mean of the four cells. Perhaps this is not what we wanted to obtain but if we estimate a model with several categorical predictors it might advisable to code even simple comparisons like the dummy coding by means of the scheme presented above, since the constant always remains the grand mean. Even though this form of a contrast is hardly ever used, it illustrates what can be done by converting a contrast scheme into a matrix which provides the coefficients for the contrast variables. Nevertheless, xi3 provides a feature to estimate this contrast by the prefix g (Table 77). The constant can be verified by:

$$\text{display}(15.788732+18.469376+15.380424+16.633238) / 4 = \mathbf{16.567943}$$

(cell means can be found in Table 59)

Table 77: xi3:regress dist1sum g.skt

g.skt		_Iskt_1-4		(naturally coded; _Iskt_1 omitted)	
Source	SS	df	MS		
Model	8546.16828	3	2848.72276	Number of obs =	4976
Residual	203717.456	4972	40.9729396	F(3, 4972) =	69.53
Total	212263.624	4975	42.666055	Prob > F =	0.0000
				R-squared =	0.0403
				Adj R-squared =	0.0397
				Root MSE =	6.401

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_Iskt_2	2.680644	.2242624	11.95	0.000	2.240991	3.120297
_Iskt_3	-.4083086	.2623456	-1.56	0.120	-.9226217	.1060046
_Iskt_4	.8445054	.2935654	2.88	0.004	.2689877	1.420023
_cons	16.56794	.096934	170.92	0.000	16.37791	16.75798

6.2 Forward Adjacent coding

Lets assume that we do not have a feature like `xi3` in STATA but nevertheless want to model forward adjacent coding as in chapter 4.1. We again use out 4 category variable `skt` and `dist1sum` as criterion. First we have to define a matrix (*contrast matrix*) which represents the contrast coding system. As before, this matrix can then be converted to the design matrix used to compute the variables to be employed as predictor variables. This *contrast matrix* where each *row* represents one contrast appears as follows:

Table 78: Contrast matrix for forward adjacent coding (each row represents a contrast)

	c1	c2	c3	c4
r1	1	-1	0	0
r2	0	1	-1	0
r3	0	0	1	-1

Thus, the first line represents the difference between category 1 and 2, the second line the difference between 2 and 3, and the 3rd line the difference between 3 and 4. Each row sums up to zero and the row containing the sum of each column also sums up to zero. This matrix can be defined using the command:

```
matrix input L = (1 -1 0 0 \
                  0 1 -1 0 \
                  0 0 1 -1)
```

In order to obtain the regression coding scheme we need to define the three necessary variables. These are calculated as was done before. Each row yield the coefficients for each contrast variable with regard to each category of a the original predictor. If we list this matrix, we obtain a matrix similar to the matrix provided in Table 56; only rows and columns are interchanged.

Table 79: Design matrix for forward adjacent coding

	a1	a2	a3
c1	.75	.5	.25
c2	-.25	.5	.25
c3	-.25	-.5	.25
c4	-.25	-.5	-.75

By means of these coefficients, we now can define the 3 necessary predictors for the contrast we want to test. Each column of Table 79 provides the coefficients to compute the design variables a1, a2 and a3:

```

gen      a1=.75      if skt == 1
replace a1=-.25     if skt > 1
gen      a2=.5      if skt < 3
replace a2=-.5      if skt > 2
gen      a3=.25     if skt < 4
replace a3=-.75     if skt == 4

```

The three variables called a1, a2 and a3 are equal to those already defined and called A_1 A_2 and A_3. Employing these predictors will provide the same results as what the prefix a from xi3 would have done for the user. Of course, all interpretations hold, too. Compare this result with the results from chapter 4.1.1. Results equal those in Table 60.

Table 80: regress dist1sum a1 a2 a3

Source	SS	df	MS			
Model	8546.16828	3	2848.72276	Number of obs =	4976	
Residual	203717.456	4972	40.9729396	F(3, 4972) =	69.53	
Total	212263.624	4975	42.666055	Prob > F	= 0.0000	
				R-squared	= 0.0403	
				Adj R-squared	= 0.0397	
				Root MSE	= 6.401	

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
a1	-2.680644	.2242624	-11.95	0.000	-3.120297	-2.240991
a2	3.088953	.2532953	12.20	0.000	2.592382	3.585523
a3	-1.252814	.3162997	-3.96	0.000	-1.872901	-.6327269
_cons	16.56794	.096934	170.92	0.000	16.37791	16.75798

6.3 Forward Helmert Coding

For sake of completeness we present the “manual” coding of the Helmert contrast. The contrast matrix L should be defined as:

```
matrix input L=(1 -.333333 -.333333 -.33333 \
                0 1 -.5 -.5 \
                0 0 1 -1)
```

If we transform this matrix to the design the usual way: $L' \cdot \text{inv}(L \cdot L')$, we obtain the matrix K :

Table 81: Design matrix for Helmert coding

	h1	h2	h3
c1	.7500015	7.500e-07	1.125e-06
c2	-.2499995	.66666642	-3.750e-07
c3	-.2499995	-.33333358	.49999963
c4	-.2499995	-.33333358	-.50000037

Table 81 shows the design which is identical to the design matrix in Table 66. Only the rows and columns are interchanged. It seems not to be necessary to compute the empirical example again.

6.4 An arbitrary coding scheme not provided by `xi3`

6.4.1 One 4-categorical predictor and 3 contrasts

In this last section we will show a contrast which has to be built by oneself, because it is not a feature of the command `xi3`. Imagine you want to compare the first category of a 4 –category variable with the second category, the first category with the average of category 2 and 3 and,

finally, the first category with the mean of all the other categories. The contrast matrix is given below.

Table 82: Contrast matrix L

1	-1	0	0
1	-.5	-.5	0
1	1/3	1/3	1/3

and read in this matrix by means of the STATA command :

```
matrix input L= (1 -1 0 0 \
                 1 -.5 -.5 0 \
                 1 -.33333 -.33333 -.33333)
```

```
matrix K= L'*inv(L*L')
```

After transforming L to K by the same procedure as before, we obtain the coding scheme :

Table 83: Design matrix for one 4 categorical variable and 3 contrasts

	u1	u2	u3
c1	-2.221e-16	-.000015	.75001875
c2	-1	-.000015	.75001875
c3	1	-2.000015	.75001875
c4	0	1.999985	-2.2499887

By means of these coefficients we create the two variables for the two contrasts the usual way by writing:

```
gen u1= 0 if skt == 1
replace u1= -1 if skt == 2
replace u1= 1 if skt == 3
replace u1= 0 if skt == 4

gen u2= -.000015 if skt == 1
replace u2= -.000015 if skt == 2
replace u2= -2.000015 if skt == 3
replace u2= 2 if skt == 4
```

```
gen u3 = .75001875      if skt !=4
replace u3= -2.2499887  if skt ==4
```

and from a regression analysis we yield:

Table 84: regress dist1sum u1 u2 u3

Source	SS	df	MS			
Model	8546.16828	3	2848.72276	Number of obs =	4976	
Residual	203717.456	4972	40.9729396	F(3, 4972) =	69.53	
Total	212263.624	4975	42.666055	Prob > F =	0.0000	
				R-squared =	0.0403	
				Adj R-squared =	0.0397	
				Root MSE =	6.401	

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
u1	-2.680644	.2242624	-11.95	0.000	-3.120297	-2.240991
u2	-1.136168	.208614	-5.45	0.000	-1.545143	-.7271922
u3	-1.03895	.2028086	-5.12	0.000	-1.436544	-.6413556
_cons	16.56795	.0969344	170.92	0.000	16.37791	16.75798

The constant is the grand mean, as before. The effect of u1 should portray the difference between the first and the second category:

```
display 15.788732-18.469376 = -2.680644
```

The second contrast u2 represents the difference between the first category and the mean of second and third category:

```
display 15.788732- ((18.469376+15.380424)/2) = 1.136168
```

The third contrast estimate the difference between the first category and the means of all other categories.:

```
display 15.788732- ((18.469376+15.380424+16.633238)/3) = 1.0389473
```

However, we could make our life much simpler adopting the u option of xi3. We could define a pointer by writing:

```
char skt[user] (1 -1 0 0 \ 1 -.5 -.5 0 \ 1 -.33333 -.33333 -.33333)
```

and then estimating model again, which yields the same results as before. Unfortunately, the output does not provide any information on how the contrasts are actually defined !! Only the labels of the 3 new variables give the necessary information. One should be careful, because

the output does not provide any variable labels, so one never knows – from the output alone – which sort of a contrast was employed.

Table 85: `xi3:regress dist1sum u.skt`

u.skt _Iskt_1-4 (naturally coded; _Iskt_4 omitted)

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_Iskt_1	-2.680644	.2242624	-11.95	0.000	-3.120297	-2.240991
_Iskt_2	-1.136168	.208614	-5.45	0.000	-1.545143	-.7271922
_Iskt_3	-1.038944	.2028077	-5.12	0.000	-1.436537	-.6413518
_cons	16.56794	.0969344	170.92	0.000	16.37791	16.75798

6.4.2 One 4-categorical variable and 3 contrasts and one dichotomous predictor with interaction.

It might be of interest to estimate the contrasts from 6.4.1 dependent on a dichotomous predictor, as we extensively have done before. We use `xi3` to construct the interaction terms and obtain the following output.

Table 86: `xi3:regress dist1sum q0rec*u1 q0rec*u2 q0rec*u3`

Source	SS	df	MS	Number of obs = 4976		
Model	20101.1504	7	2871.59292	F(7, 4968) = 74.24		
Residual	192162.473	4968	38.680047	Prob > F = 0.0000		
				R-squared = 0.0947		
				Adj R-squared = 0.0934		
				Root MSE = 6.2193		

dist1sum	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
q0rec	-2.798346	.195587	-14.31	0.000	-3.181783	-2.414909
u1	-1.244415	.3214451	-3.87	0.000	-1.874589	-.6142403
_Iq0Xu1	-.7341072	.4557537	-1.61	0.107	-1.627586	.1593714
u2	.1530006	.3298555	0.46	0.643	-.4936619	.799663
_Iq0Xu2	-1.337024	.4243911	-3.15	0.002	-2.169018	-.5050305
u3	.4278688	.3136252	1.36	0.173	-.1869751	1.042713
_Iq0Xu3	-1.660981	.4090057	-4.06	0.000	-2.462812	-.8591487
_cons	18.02827	.1413196	127.57	0.000	17.75122	18.30532

`_Iq0Xu1`, `_Iq0Xu2` and `_Iq0Xu3` are nothing but the products of `q0` and one of the `u` terms. If one does not have `xi3`, he or she must compute the three interaction variables “by hand”. Again, we should ask what the constant really represents. Since the dichotomous

variable q_0 is dummy coded it should be the grand mean conditioned on $q_0 = 0$. We calculate this mean using the information from the table in section 4.1.2 .

$$\text{display } (18.349177+19.593592+16.798762+17.371571)/4 = \mathbf{18.028275}$$

If we calculate the grandmean for the other row ($q_0=1$), we obtain:

$$\text{display } (14.305085+16.283607+14.694611+15.636364)/4 = \mathbf{15.229917}$$

and the difference is -2.798346 . This difference is represented by the conditional effect called $q_0\text{rec} = -2.798346$ (First row of Table 86) .

6.4.2.1 First contrast u_1

The effect u_1 represents the difference between the first category and the second conditioned on $q_0=0$ (schizophrenia) .

$$\text{display } (18.349177-19.593592) \quad \mathbf{-1.244415}$$

and conditioned on $q_0=1$ (depression) this difference is :

$$\text{display } (14.305085-16.283607) \quad \mathbf{-1.978522}$$

We see the effect u_1 is not equal for the 2 groups , but rather the difference is:

$$\text{display } 1.244415-1.978522 \quad \mathbf{-.734107}$$

which is portrayed by the interaction between u_1 and q_0 , called $_Iq_0Xu_1 = (.7341072)$

Therefore, the interaction term reports the difference between the 2 differences. This interaction is not significant, so we might conclude that the difference between the first and the second category does not depend on q_0 (vignette) . In other words, the difference in social distance for those who labelled the person in the cases stories explicitly correct, or only as mentally ill, is significantly different, but this difference is equal for both the schizophrenic episode and the major depression.

6.4.2.2 Second contrast u_2

We should briefly have a look at the second contrast u_2 , which represents the difference between the first category and the average of 1 & 2:

$$18.349177 - ((19.593592 + 16.798762) / 2) = \mathbf{0.153}$$

We see from the table that this difference is not significant as far as the schizophrenia ($q_0=0$) is concerned.

$$(14.305085 - ((16.283607 + 14.694611) / 2)) = \mathbf{-1.184024}$$

The difference not only turned out to be the other way round; if the schizophrenia vignette is presented, the “correct diagnosis” exerts a more pronounced (although nonsignificant) social distance. If the case story of a major depression was presented, people show considerably less social distance, if they label the story as depression, compared to the average of the two following categories (mental illness and illness). The interaction effect $I_{q_0 \times u_2}$ tells us that the difference is exactly $\mathbf{-1.337024}$ and highly significant.

Study suggestions 7: Please interpret the results for the third contrast u_3 .

Of course, we can test whether these 3 interaction terms are simultaneously significant (different from zero):

Table 87: **test** **$_I_{q_0 \times u_1}$** **$_I_{q_0 \times u_2}$** **$_I_{q_0 \times u_3}$**

- (1) $_I_{q_0 \times u_1} = 0$
- (2) $_I_{q_0 \times u_2} = 0$
- (3) $_I_{q_0 \times u_3} = 0$

$$F(3, 4968) = 7.40$$
$$\text{Prob} > F = 0.0001$$

Comparing the 2 adjusted R^2 , we observe an increase of about 5.3%, so we might conclude that the case stories have an effect on the contrasts we wanted to estimate.

7 Reference List

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8 Appendix

Vignette schizophrenia (q0rec = 0)

Imagine that you hear the following about an acquaintance with whom you occasionally spend your leisure time:

Within the past months, your acquaintance appears to have changed. More and more, he retreated from his friends and colleagues, up to the point of avoiding them. If someone managed to involve him in a conversation, he would address only one single topic: the question as to whether some people had the natural gift of reading other people's thoughts. This question became his sole concern. In contrast with his previous habits, he stopped taking care of his appearance and looked increasingly untidy. At work, he seemed absent-minded and frequently made mistakes. As a consequence, he has already been summoned to his boss. Finally, your acquaintance stayed away from work for an entire week without an excuse. Upon his return, he seemed anxious and harassed. He reports that he is now absolutely certain, that people cannot only read other people's thoughts, but that they also directly influence them. He was however unsure who would steer his thoughts. He also said that, when thinking, he was continually interrupted. Frequently, he would even hear those people talk to him, and they would give him instructions. Sometimes, they would also talk to each other and make fun of whatever he was doing at the time. The situation was particularly bad at his apartment, he claimed. At home, he would really feel threatened, and would be terribly scared. Hence he had not spent the night at his place for the past week, but rather he had hidden in hotel rooms and hardly dared to go out.

Vignette major depressive disorder (q0rec = 1)

Imagine that you hear the following about an acquaintance with whom you occasionally spend your leisure time:

Within the past two months, your acquaintance has changed in his nature. As opposed to previously, he is down and sad without being able to make out a concrete reason for his feeling low. He appears serious and worried. There is nothing anymore that will make him laugh. He hardly ever talks, and if he says something, he speaks in a low tone of voice about the worries he has with regard to his future. Your acquaintance feels useless and has the impression to do everything wrong. All attempts to cheer him up have failed. He lost all interest in things and is not motivated to do anything. He complains of often waking up in the middle of the night and not being able to get back to sleep. Already in the morning, he feels exhausted and without energy. He says that he encounters difficulty in concentrating on his job. In contrast with previous times, everything takes him very long. He hardly manages his workload. As a consequence, he has already been summoned to his boss.